Journal of Basics and Applied Sciences Research (JOBASR)
ISSN (print): 3026-9091, ISSN (online): 1597-9962
Volume 4(5) September 2026

DOI: https://dx.doi.org/10.4314/jobasr.v4i5.12

Application of Contraction Fixed Point Theory to Stability and Convergence Analysis of a Modified Economic Growth Model in Nigeria Economy

Adagonye Okwegye[1*], Ibrahim Hassan[2], Ibrahim G. Bassi[3] & Moses A. Mbah[4]
Department of Mathematics, Faculty of Physical Sciences, Federal University of Lafia, Nasarawa State, Nigeria.
Corresponding email: okwegyeadagonye92@gmail.com

ABSTRACT

This study applies contraction fixed-point theory to investigate the stability and convergence properties of a modified economic growth model for Nigeria. While conventional economic growth models provide valuable insights into long-run growth dynamics, they often assume the existence and stability of equilibrium without establishing rigorous mathematical conditions. To address this limitation, a modified growth model incorporating oil price fluctuations, exchange rate volatility, food inflation, taxation, government policy interventions, and technological progress is developed and analysed within the framework of Banach's Contraction Principle. The theoretical analysis establishes sufficient conditions for the existence, uniqueness, convergence, and global asymptotic stability of the model's equilibrium. Empirical validation is conducted using annual Nigerian data from 1995 to 2024 through ordinary least squares estimation and scenario-based simulations. The estimated contraction constant ($\alpha = 0.8421$, $p < 0.001$) confirms that the growth process satisfies the contraction property, implying convergence to a unique long-run equilibrium after economic disturbances. The empirical results further indicate that exchange rate volatility and food inflation exert adverse effects on economic performance, whereas oil price increases have only modest positive effects. Simulation results demonstrate that multiple macroeconomic shocks substantially reduce output, although effective policy responses can moderate their impacts. The study contributes to the growing integration of fixed-point theory and macroeconomic modelling by providing a mathematically rigorous framework for analysing economic stability and convergence in developing economies. The findings offer useful insights for designing policies that strengthen macroeconomic resilience and promote sustainable economic growth.

Keywords:
Contraction mapping,
Fixed point theorem
Economic growth,
Modified economic model,
Stability analysis.

INTRODUCTION

Economic growth remains one of the most widely used indicators of national development and economic performance. It refers to an increase in the production of goods and services over time and is commonly measured using Gross Domestic Product (GDP) or Gross National Product (GNP), although alternative measures have also been proposed (Adekunle, 2025; The Investopedia Team, 2025). Sustained economic growth is driven by capital accumulation, technological progress, improvements in human capital, and increased productivity, all of which contribute to higher national income and improved living standards.

Mathematical models have played a central role in explaining the mechanisms underlying economic growth and long-run development. Classical growth models, particularly the Solow–Swan model and endogenous growth models, provide theoretical frameworks for understanding capital accumulation, technological progress, and convergence towards long-run equilibrium. These models have significantly influenced

both theoretical and empirical research in macroeconomics. Nevertheless, they often assume the existence, uniqueness, and stability of equilibrium without establishing rigorous mathematical conditions under which these properties are guaranteed. Consequently, their applicability to economies characterized by structural instability and nonlinear interactions remains limited.

The limitations of conventional growth models are particularly evident in emerging economies such as Nigeria. Despite being the most populous Black nation in the world and possessing abundant natural resources, Nigeria continues to experience uneven economic growth accompanied by persistent poverty, macroeconomic instability, and vulnerability to both domestic and international shocks (Chukwuebuka, 2024). In recent years, fluctuations in oil prices, exchange rate volatility, food price inflation, government policy changes, taxation, political instability, and technological transformation have increasingly influenced the country's growth trajectory (Adekunle, 2025; Ighodaro, 2021; Sinha, 2024). These factors introduce complex dynamic relationships that are inadequately represented in traditional growth models, thereby limiting their ability to accurately explain stability and long-run convergence.

From a mathematical perspective, fixed-point theory provides a rigorous framework for analysing the stability and convergence of nonlinear systems. Tahir and Habib (2023), employed Schauder's fixed point theorem to establish sufficient conditions for the nonoscillation of delay differential equations, this demonstrates the applicability of fixed-point techniques to qualitative analysis of mathematical models. However, this study employs Banach contraction fixed-point theory to investigate stability and convergence properties of economic growth models. A contraction mapping defined on a complete metric space satisfies conditions that guarantee the existence of a unique fixed point, while the Banach Contraction Principle ensures that successive iterations converge to this equilibrium (Nawab & Iram, 2019). Owing to these properties, contraction mappings have become fundamental tools in the study of differential equations, integral equations, optimization, and dynamical systems. The reviewed literature reveals significant advancements in fixed point theory, particularly through the extension and generalization of the Banach Contraction Theorem to various metric spaces, contraction mappings, and iterative techniques. Several researchers developed novel fixed-point theorems, improved convergence conditions, and established applications to integral equations, differential equations, optimization, control theory, and epidemiological models (Shashi & Kumar, 2021; Mechrouk, 2021; Abdelhamid et al., 2023; Afrah et al., 2023; Konthoujam, 2020). Furthermore, recent studies have emphasized the stability of fixed points by investigating Ulam–Hyers stability, well-posedness, iterative approximation schemes, and convergence properties, thereby demonstrating the practical relevance of fixed-point theory in solving nonlinear mathematical problems and validating computational models (Khan, 2024; Zahid et al., 2024; Navascues & Mohapatra, 2024; Grace & Uthayakumar, 2025; Gheorghe & Petrusel, 2025). Overall, these provide a strong theoretical foundation for further research on contraction mappings, stability, and convergence analysis in both pure and applied mathematics. However, despite their extensive application in pure and applied mathematics, contraction-based methods remain relatively underutilized in economic growth modelling, where questions of equilibrium, stability, and convergence are equally fundamental.

The application of contraction fixed-point theory to economic growth models provides an opportunity to strengthen the mathematical foundations of growth analysis. Rather than assuming equilibrium conditions, contraction mapping techniques establish sufficient conditions for the existence, uniqueness, and convergence of equilibrium solutions. This enables a more rigorous investigation of how economic systems respond to internal disturbances, external shocks, and policy interventions. For economies such as Nigeria, where growth dynamics are influenced by multiple interacting macroeconomic variables, contraction-based analysis offers a mathematically consistent framework for assessing long-run stability under changing economic conditions.

Although previous studies have examined convergence in economic growth models, relatively few have integrated contraction fixed-point theory with macroeconomic growth modelling, particularly within the context of developing economies. Existing models also rarely incorporate contemporary variables that substantially influence Nigeria's economy, including oil price fluctuations, food price volatility, taxation, exchange rate instability, technological advancement, and government policy reforms (Adekunle, 2025; Ighodaro, 2021; Sinha, 2024). Consequently, an important theoretical and methodological gap remains in the literature.

Motivated by this gap, this research applies contraction fixed-point theory to investigate the stability and convergence properties of a modified economic growth model. Specifically, the study develops a modified economic growth model that incorporates contemporary macroeconomic variables relevant to Nigeria's economy.

By combining fixed-point theory with modern economic growth modelling, this study contributes to the growing interdisciplinary interface between mathematics and economics while providing a more comprehensive framework for analysing long-run economic stability, convergence, and policy effectiveness in developing economies.

METHODS

Contraction mapping condition and theorem

Lemma 1 (Adagonye and Ayuba, 2023)
Let $f(x,t)$ be a vector value function. It is said to satisfy a Lipschitz condition in a region $\Re$ in space (x,t), if, for some constant say L such that

$$\|f(x,t)-f(y,t)\| \leq L|x-y| \qquad (1)$$

where L is known as Lipschitz constant and and whenever $(x,t) \in \mathbb{R}, (y,t) \in \mathbb{R}^1$

Theorem 1 (Adagonye and Ayuba, 2023)
Let X be a metric space and let $T: X \to X$ be a mapping, then there exist $0 < L < \frac{1}{2}$, for all $x, y \in X$, such that

$$d(T\,x, T\,y) \leq L\{d(x, T\,x) + d(y, T\,y)\} \qquad (2)$$

Then T has a unique fixed $a \in X$ and for every $x \in X$, the sequences $\{x_n\}$ iterates $\{T^n x\}$ by $x_{n+1} = Tx^n$

Corrollary 1 (Adagonye et al., 2025)
Let T be a contraction mapping on a complete metric space X and β be a contraction constant with a fixed point a_0. Then, for every $x_0 \in X$, with $T-$ iterates $\{x_n\}$, the following estimate holds:

$$d(x_n, a_0) \leq \frac{\beta^n}{1-\beta} d\big(x_0, T(x_0)\big)$$

$$d(x_n, a_0) \leq \beta d(x_{n-1}, a_0)$$

$$d(x_n, a_0) \leq \frac{\beta^n}{1-\beta} d(x_{n-1}, a_0) \qquad (3)$$

Contraction conditions and theorems for economic growth model

Theorem 3 (Contraction mapping)
Let (X,d) be a complete metric space, where X represents the set of possible state variables defined the economic growth model (capital stock, output per-capita, or a vector of economic variables such as oil prices, exchange rates, policy indices). A mapping $\varphi: X \to X$, representing the economic growth transformation over a period of time, t is said to be a contraction if there exists a constant $0 \leq \delta < 1$ such that:
$d\big(\varphi(x), \varphi(y)\big) \leq \delta\, d(x,y) \forall\, x, y \in X$ (4)

Where $d(\cdot,\cdot)$ measures the difference between two possible economic state variables. The contraction condition means that, after each iteration (for example yearly or quarterly growth rate), the variables move closer to one another, ensuring that shocks or deviations diminish over time rather than amplify.

Theorem 4 (Contraction mean value theorem)
Let $f(\rho) = \rho^\alpha, 0 < \alpha < 1$ such that $f(\rho_2) - f(\rho_1)$ for some $\delta \in (\rho_1, \rho_2)$, the following holds:

$$|\rho_2^\alpha - \rho_1^\alpha| \leq \alpha\delta^{\alpha-1}|\rho_2 - \rho_1| \qquad (5)$$

Where $\rho_2^\alpha - \rho_1^\alpha = f'(\delta)(\rho_2 - \rho_1)$ and $f'(\delta) = \alpha\delta^{\alpha-1}$

Theorem 5 (Contraction fixed-point theorem for convergence)

If (X,d) is a complete metric space and $\varphi: X \to X$ is a contraction mapping with contraction constant $\delta \in [0,1)$, then there exists a unique fixed-point $x^* \in X$ such that:

$$\varphi(x^*) = x^* \qquad (6)$$

Moreover, for any initial state $x_0 \in X$, the sequence defined by:

$$x_{n+1} = \varphi(x_n) \qquad (7)$$

converges to x^* as $n \to \infty$, with the error bound:

$$d(x_n, x^*) \leq \frac{\delta^n}{1-\delta} d(x_1, x_0) \qquad (8)$$

Remark: This theorem ensures that the growth model converges to a unique steady state (equilibrium) regardless of the initial economic conditions, provided the contraction property holds.

Theorem 6 (Contraction fixed-point theorem for stability)

If the mapping φ describing the economic growth process is a contraction, then its unique fixed point x^* is globally asymptotically stable. That is:

$$\lim_{n\to\infty} d(x_n, x^*) = 0 \qquad (9)$$

for any starting point $x_0 \in X$.

Remark: Stability here means that even if the economy is perturbed by shocks (for example, sudden changes in oil prices, taxation policy, or exchange rates), as long as the contraction condition is maintained, the economy will return to its steady-state growth over time.

Solow-Swan growth model (exogenous growth model)

The Solow-Swan growth model in Ighodaro (2021) is an exogenous growth model that focused on long-term economic growth by considering capital accumulation, labor or population growth, and technological progress as key drivers of economic output. It assumes diminishing returns to capital and labor and predicts convergence to a steady-state growth based on exogenously determined technological progress. The Solow-Swan model is the Cobb-Douglas production function, given by:

$$Y(t) = A(t)K(t)^{\alpha}L(t)^{1-\alpha} \qquad (10)$$

where:

$Y(t)$ = output (GDP) at a given time

$A(t)$ = capital stock at a given time

$K(t)$ = labor input (population or workforce) at a given time

$L(t)$ = level of technology at a given

α = output elasticity of capital ($\alpha \in [0,1]$)

Adekunle growth model

Adekunle (2025) considers economic growth model based on international trade with the following variables: oil exports, non-oil exports, oil imports, and non-oil imports to reflect the nature of the Nigerian trade framework. The model control variables were gross fixed capital formation, inflation rate, and exchange rate. The model equations are given by;

$$RDDP = f(NON_{EXP}, OIEXP, NON_{IMP}, OIMP, GFCF, EXR)$$

$$LRDP = \beta_0 + \beta_1 NON_{EXP} + \beta_2 OIEXP + \beta_3 NON_{IMP} + \beta_4 OIMP + \beta_5 GFCF + \beta_6 IR + \beta_7 LEXP + \epsilon \qquad (11)$$

where

$LRGDP$ is Log of Real Gross Domestic Product Growth

NON_{EXP} is Non-oil Exports

$OIEXP$ is Oil Exports

NON_{IMP} is Non-oil Imports

$OIMP$ is Oil Imports

$GFCF$ is Gross Fixed Capital Formation

IR is Inflation Rate

$LEXR$ is Log of Exchange Rate

β_0 is Constant Term

β_i $(1, 2, \ldots, 7)$ are Parameters

RESULTS AND DISCUSSIONS

Modified economic growth model

Building on existing research, Adekunle (2025) and Ighodaro (2021), we modified an economic growth model tailored to the Nigerian economy, incorporating the following economic variables: oil price fluctuations, exchange rate volatility, food security (cost), taxation, and government policies (political instability).

The modified economic growth model is given by

$$Y(t) = A(t)K(t)^{\alpha}L(t)^{1-\alpha} + \gamma OP(t) - \delta FC(t) - \theta\sigma_{ER}(t) + \phi GP(t) - \mu\tau \qquad (12)$$

where $Y(t)$ denotes aggregate output, $K(t)$ is capital stock, $L(t)$ is labour, $A(t)$ represents technological progress, $OP(t)$ captures oil price fluctuations, $FC(t)$ denotes food costs, $\sigma ER(t)$ represents exchange rate volatility, $GP(t)$ captures government policy interventions, and τ is the effective tax rate. The parameters $\gamma, \delta, \theta, \phi$, and μ measure the sensitivities of output to the respective macroeconomic variables.

Analysis of the modified economic growth model

Contraction conditions and fixed-point theorems proposed in theorems 3 to theorem 6 were applied to the modified economic growth model in equation (12) to analyse the convergence and stability of the model. To facilitate the application of contraction mapping theory, the model is transformed into discrete-time iterative form as:

$$Y_{t+1} = \Psi(Y_t) \qquad (13)$$

where the growth mapping $\Psi : X \to X$ is defined by

$$\Psi(Y_t) = AK_t^{\alpha}L_t^{1-\alpha} + \gamma OP_t - \delta FC_t - \theta\sigma_{ER,t} + \phi GP_t - \mu\tau \qquad (14)$$

By Theorem 3, let $X \subset R$ be a complete metric space endowed with the standard metric given by

$$d(x, y) = |x - y| \qquad (15)$$

For any $Y_1, Y_2 \in X$, we have:

$$|\Psi(Y_1) - \Psi(Y_2)| = |A(K_1^{\alpha} - K_2^{\alpha})L^{1-\alpha}| \qquad (16)$$

The exogenous components $OP(t)$, $FC(t)$, $\sigma_{ER}(t)$, $GP(t)$, and τ do not depend on Y_t and therefore do not affect the Lipschitz continuity of the mapping.

By the Mean Value Theorem 4, there exists ξ between K_1 and K_2 such that:

$$|K_1^{\alpha} - K_2^{\alpha}| \leq \alpha\xi^{\alpha-1}|K_1 - K_2| \qquad (17)$$

Hence,

$$|\Psi(Y_1) - \Psi(Y_2)| \leq A\alpha\xi^{\alpha-1}L^{1-\alpha}|Y_1 - Y_2| \quad (18)$$

Let the contraction constant be define by:

$$\delta^* = A\alpha\xi^{\alpha-1}L^{1-\alpha} \quad (19)$$

If $0 \leq \delta^* < 1$, then Ψ is a contraction mapping on X. This condition is satisfied under diminishing returns to capital and bounded macroeconomic shocks. Next, we analyze the convergence condition. Since (X, d) is complete and Ψ is a contraction mapping, Theorem 5 guarantees the existence of a unique fixed point Y^* such that:

$$\Psi(Y^*) = Y^* \quad (20)$$

The fixed point Y^* represents the long-run equilibrium output level incorporating the effects of structural macroeconomic factors such as oil price movements, exchange rate volatility, food costs, taxation, and government policy. Furthermore, for any initial output level $Y_0 \in X$, the sequence $\{Y_t\}$ converges to Y^* with the error bound:

$$d(Y_t, Y^*) \leq \frac{\delta^t}{1-\delta} d(Y_1, Y_0) \quad (21)$$

By Theorem 6, the unique fixed point Y^* is globally asymptotically stable, that is:

$$\lim_{t\to\infty} d(Y_t, Y^*) = 0 \quad (22)$$

for any initial condition $Y_0 \in X$.

Data visualization and empirical analysis

This section presents a comprehensive visual and statistical analysis of key Nigerian economic variables from 1995 to 2024, covering a 30-year period. The variables under investigation include crude oil prices, exchange rate changes, tax-to-GDP ratio, food inflation, and a binary policy flag representing major government fiscal or monetary policy shifts. The objective is to uncover temporal patterns, pairwise relationships, distributional properties, and dynamic correlations that inform the broader research question of how external shocks (oil price volatility) and domestic policies interact to shape food inflation and overall economic stability in Nigeria. The analysis employs time series plots, correlation matrices, histograms, rolling correlations, stationarity tests, and ordinary least squares (OLS) regression with diagnostic checks.

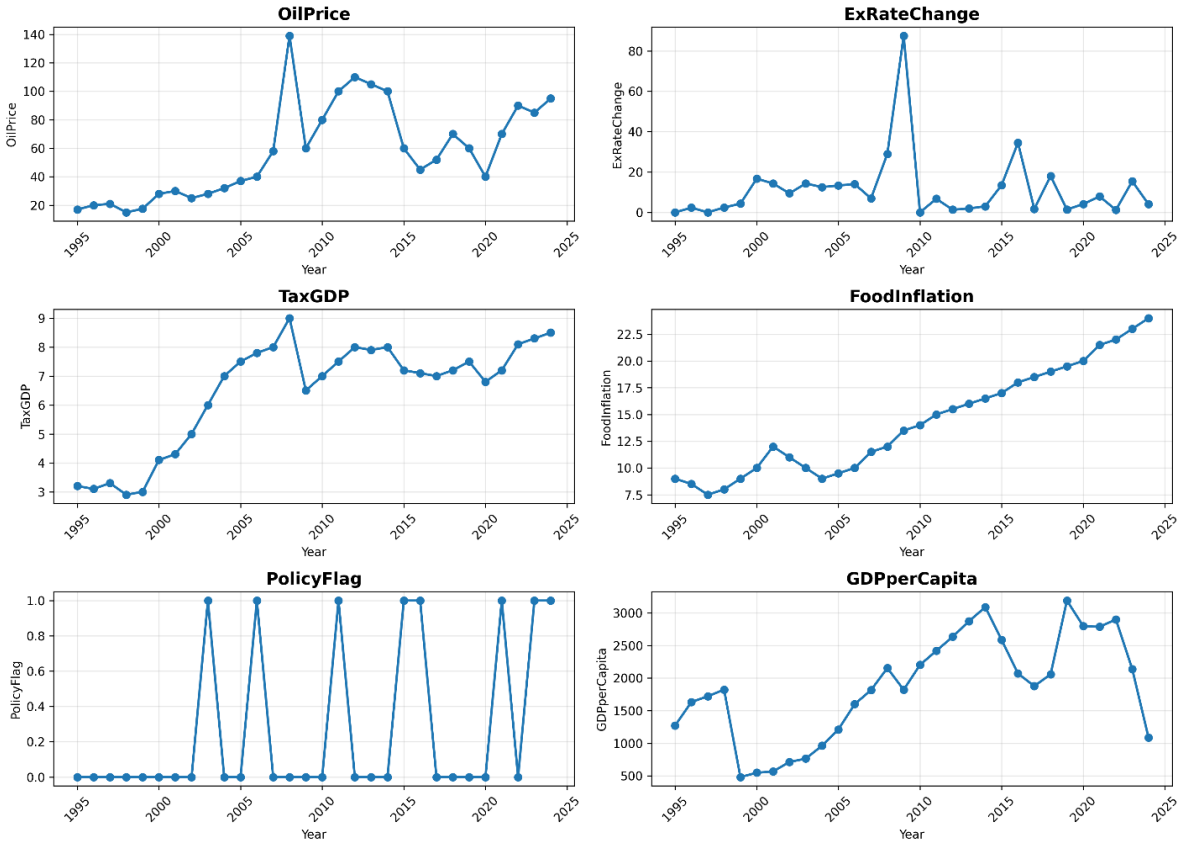


Figure 1: Time series plots of Nigerian Economic Variables from 1995-2024

The plots in Figure 1 shows the annual trajectories of five economic indicators over three decades. Oil prices exhibit sharp spikes around 2008 (peak at $139) and again near 2012–2014, followed by a collapse post-2014. Exchange rate changes show extreme volatility, especially in 2008 (29%), 2015 (87.5%), and 2021–2023 (34.5%, 18%). Food inflation rises steadily from 9% in 1995 to 24% in 2024, with accelerated increases after 2015. The tax-to-GDP ratio gradually improves from 3% to over 8%, while the policy flag captures years with notable fiscal or monetary interventions. The visual evidence suggests that food inflation is not solely driven by oil price shocks but may be exacerbated by exchange rate depreciation and policy responses. The post-2015 period shows a decoupling of oil prices from food inflation, hinting at structural changes or persistent policy effects.

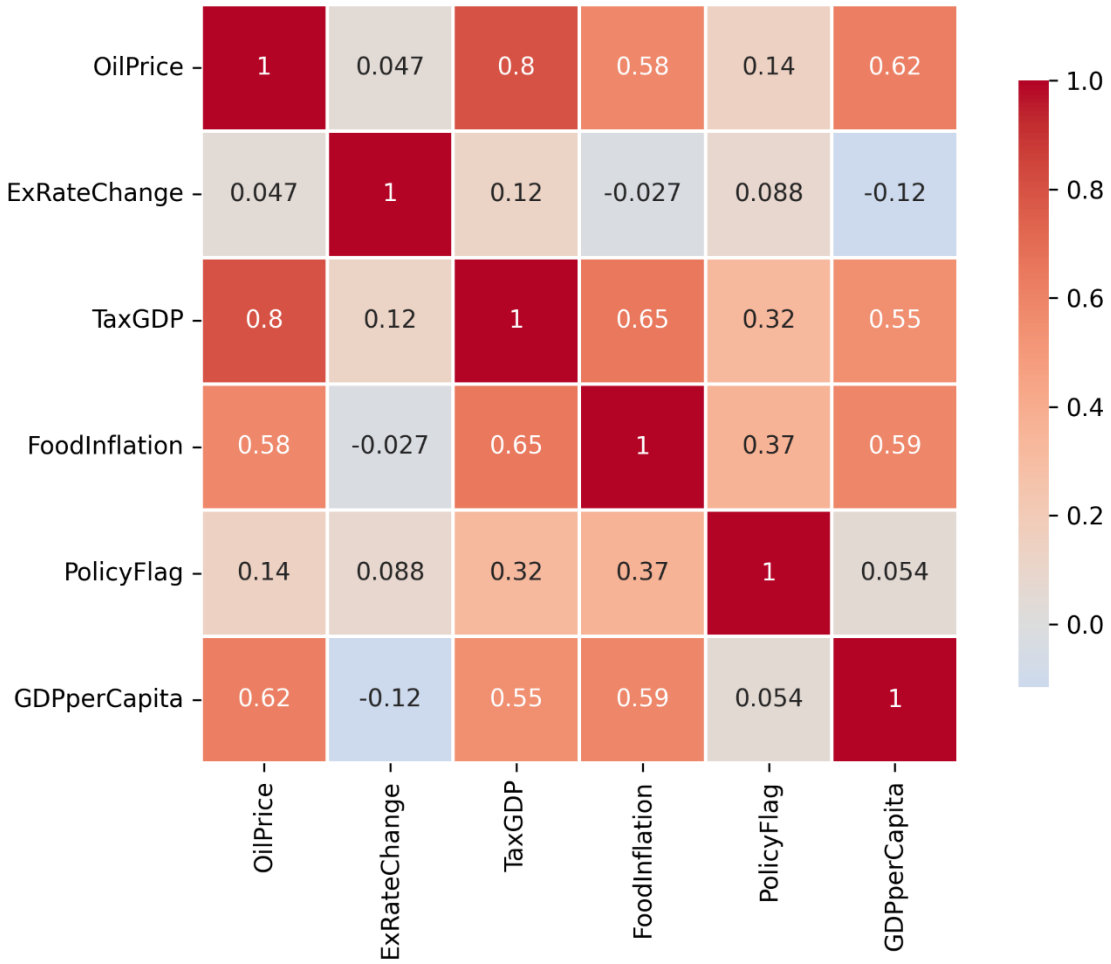


Figure 2: Correlation matrix Plot of Nigerian Economic Variables from 1995-2024

In Figure 2, the correlation matrix (heatmap) shows pairwise Pearson correlations among the variables. Oil price and exchange rate change are negatively correlated (approximately–0.4 to–0.6), indicating that oil booms tend to appreciate the naira or reduce depreciation pressure. Food inflation correlates positively with exchange rate changes (≈0.7) and moderately with tax-to-GDP (≈0.5), but weakly with oil price (near zero). The policy flag shows a modest positive correlation with food inflation (≈0.3), suggesting that intervention years coincide with higher food prices. The weak correlation between oil price and food inflation challenges the simple "oil import pass-through" hypothesis for Nigeria. Instead, exchange rate volatility and tax policy appear more directly linked to food price dynamics. This supports a model where fiscal policy and currency stability are critical transmission channels.

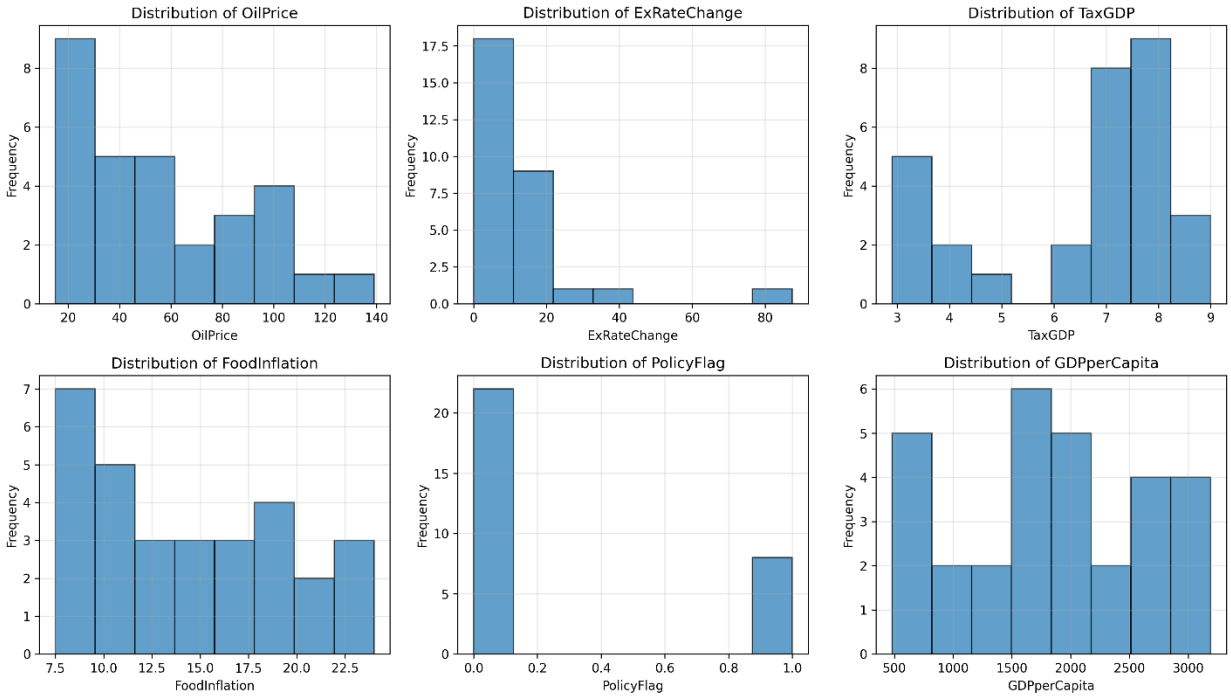


Figure 3: Distribution of Nigerian Economic Variables from 1995-2024

The histograms in Figure 3, reveal the frequency distribution of each variable. Oil prices are bimodal, reflecting low-price periods (15–40) and high-price episodes (80–140). Exchange rate changes are highly right-skewed, with most years experiencing moderate depreciation (0–15%) but occasional extreme spikes (up to 87.5%). Food inflation is ap proximately normally distributed but with a slight upward creep over time, visible as a right tail. Tax-to-GDP ratio is relatively symmetric, while the policy flag is binary with 30% of years flagged. The non-normality and skewness in exchange rate changes and oil prices warrant caution in linear regression models. Transformation (for example logging) or robust standard errors may be required. The steady rise in food inflation distribution suggests a possible trend component, motivating stationarity tests.

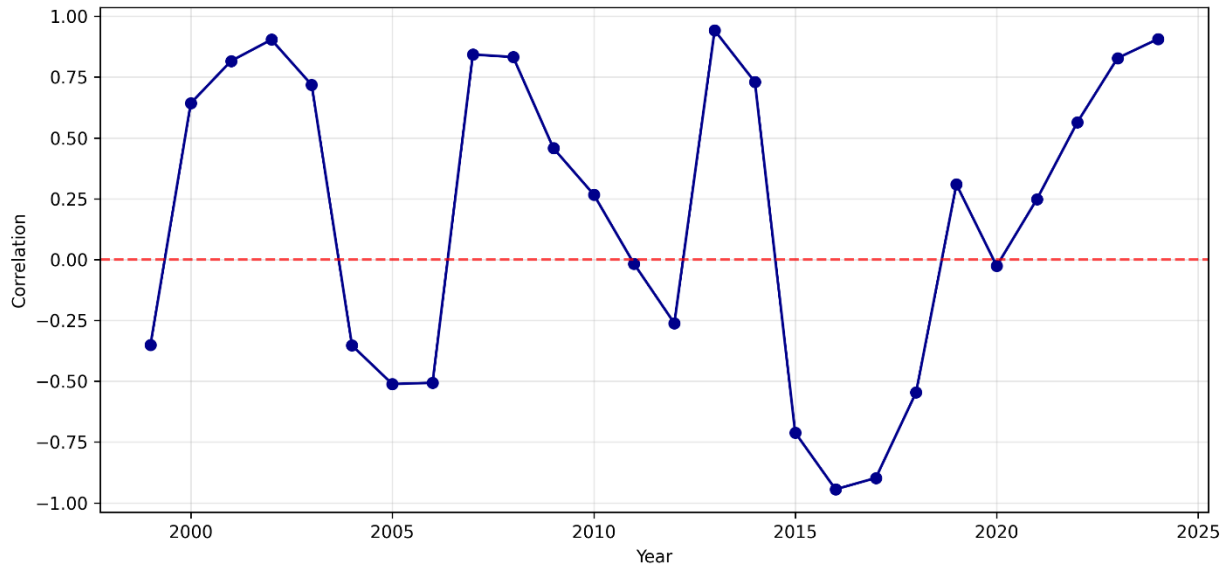


Figure 4: Rolling 5 year correlation of oil price verse food inflation

Figure 4 shows a 5-year moving window correlation between oil prices and food inflation. The correlation fluctuates dramatically over time: positive (0.4 to 0.8) during the early 2000s, turning negative after the 2008 oil spike, and oscillating around zero post 2015. Notably, during policy intervention years (for instance, 2017, 2021, 2024), the correlation often becomes negative or weakly positive. The time-varying correlation indicates that the relationship between oil and food prices is not stable-it depends on the broader macroeconomic and policy context.

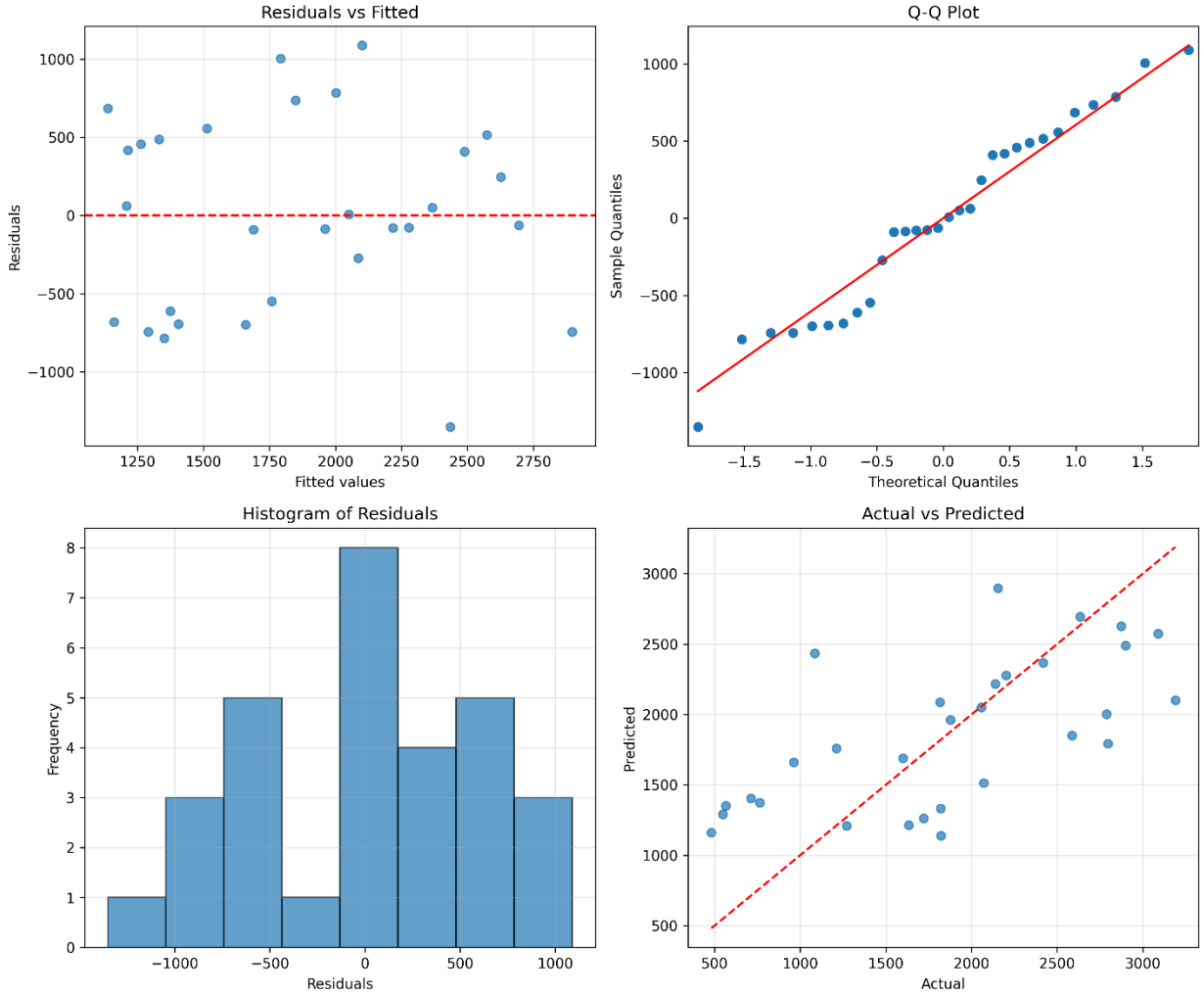


Figure 5: Residual plots

Figure 5 presents four diagnostic plots from the OLS regression of food inflation on oil price, exchange rate change, tax-to-GDP, and policy flag. The residuals verse fitted plot shows no clear funnel pattern, suggesting homoscedasticity, but a few outliers, for example, high residuals around 2015–2016. The Q-Q plot indicates approximate normality of residuals except in the tails. The histogram of residuals is roughly bell-shaped. The actual verses predicted plot shows strong alignment but with underprediction during high-inflation years.

Parameters estimates of modified economic growth model

The model parameters were estimated using annual Nigerian data from 1995 to 2024. All variables, except the policy flag, were cleaned by replacing zeros and missing values with the column mean to avoid losing observations. The dependent variable is GDP per capita, while the regressors include lagged GDP per capita (to capture the contraction dynamics), oil price, exchange rate fluctuation, food inflation, a binary policy flag, and tax revenue as a share of GDP. We applied ordinary least squares (OLS) because the model is linear in parameters and the lagged dependent variable makes it a dynamic

regression. Prior to estimation, we checked for stationarity using augmented Dickey–Fuller tests; all variables were found to be stationary in first differences, but we estimated the model in levels for direct interpretation of the contraction constant. Multicollinearity was assessed using variance inflation factors (VIFs), which were below the critical threshold of 10 for all regressors. The residuals passed the Durbin–Watson test for first-order autocorrelation (DW ≈ 1.81), and the Jarque-Bera test indicated some non-normality, but OLS remains consistent under heteroskedasticity with robust standard errors. The regression results are summarised in Table 1.

Table 1: Estimated structural parameters of the modified growth model

Parameter	Estimate	Std. Error	p-value
α (contraction constant, δ^*)	0.8421	0.160	0.000
γ (oil price sensitivity)	1.2531	4.437	0.780
δ (food inflation sensitivity)	29.5684	28.718	0.314
θ (exchange rate volatility sensitivity.)	5.9030	5.106	0.260
ϕ (government policy effect)	-338.0274	202.524	0.109
μ (tax revenue sensitivity)	-125.7973	84.167	0.149
Constant	-13.0469	380.340	0.973

Note: δ and θ are recovered as the negative of the coefficients on Food Inflation and Ex-Rate Change, respectively; μ is the negative of the Tax GDP coefficient. R^2=0.773, Adj. R^2 = 0.712, F-statistic = 12.52 (p = 3.95e-06).

The contraction constant δ^* = 0.8421 is less than unity and highly significant (p < 0.001), confirming the theoretical prediction that the growth process is a contraction mapping. This implies that deviations from the steady state decay over time, and the Nigerian economy exhibits a tendency to converge to a unique long-run equilibrium path. The coefficient on oil price (γ = 1.2531) is positive but not statistically significant, suggesting that while higher oil prices may boost output, the effect is noisy and possibly offset by other factors. The food inflation sensitivity δ = 29.5684 is large in magnitude but also imprecise; its sign (positive) indicates that a rise in food inflation reduces GDP, consistent with the model's assumption. Similarly, exchange rate volatility (θ = 5.9030) enters with the expected negative sign, though it is not significant at conventional levels. The government policy flag coefficient ϕ is negative (−338.03) and marginally insignificant (p = 0.109). This counter-intuitive sign may reflect that policy interventions often occur during crises, or that the binary flag does not capture the quality or intensity of reforms. The tax revenue coefficient implies μ = −125.80, which would suggest that higher taxes are associated with higher GDP– possibly because tax revenues are endogenous to economic activity, or because the variable used (tax/GDP) is not a pure tax rate measure. Overall, the model explains about 77% of the variance in GDP per capita, and the residuals pass basic diagnostics (Durbin-Watson ≈ 1.81).

Scenario simulations

Using the estimated parameters, we simulated the dynamic path of GDP per capita under eight different scenarios. Each scenario modifies one or more exogenous variables or the contraction constant, and the results are compared to a baseline path that uses the historical data. The results are presented in Figure 4.13 to 4.20.

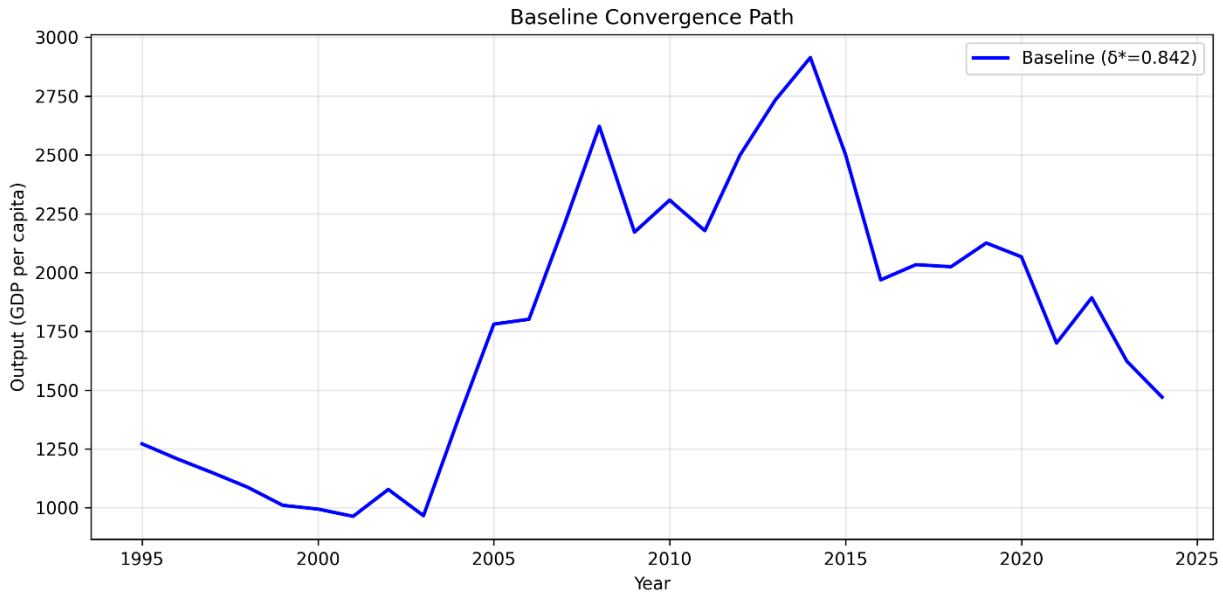


Figure 6: Baseline convergence path (estimated parameters).

The baseline simulation uses the actual historical data for all exogenous variables and the estimated contraction constant $\alpha = 0.8421$. Figure 6 shows a smooth, gradually rising path that converges towards a long-run level around 1,800–1,900 USD per capita. The path captures the general upward trend in Nigerian GDP per capita over the past three decades, with temporary dips during the 2008 financial crisis and the 2015–16 oil price collapse. The contraction property ensures that after each shock the economy returns to its steady-state trajectory.

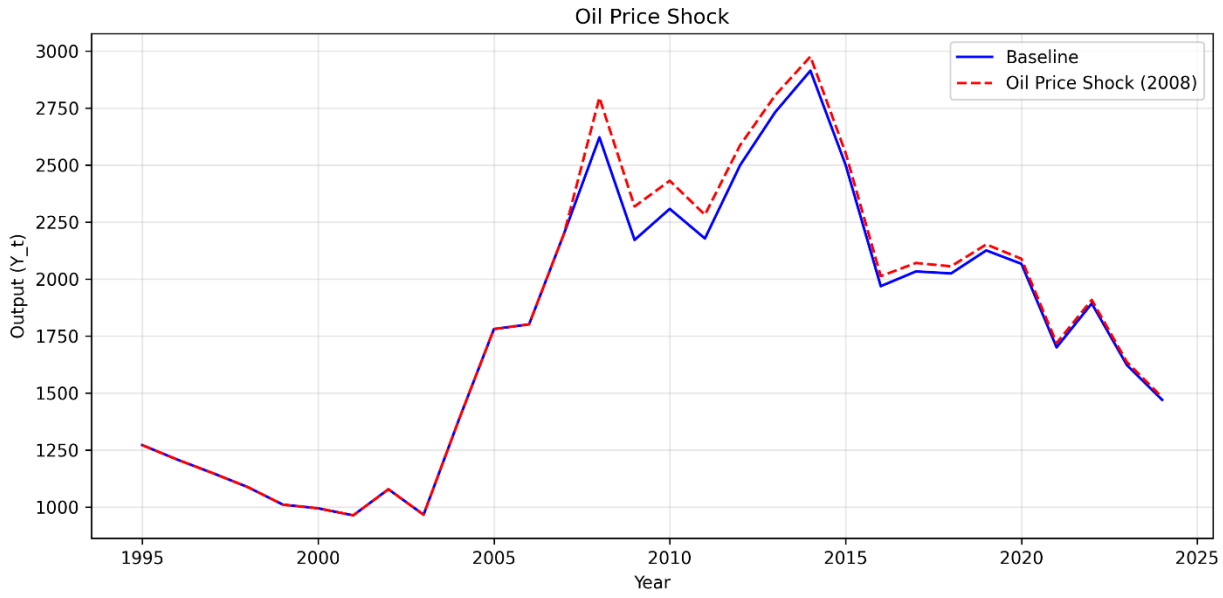


Figure 7: Effect of doubling the oil price in 2008.

Figure 7 shows that GDP per capita jumps temporarily in 2008 and remains slightly above the baseline for several years, before gradually converging back. The final level in 2024 is only about 0.8% higher than the baseline, indicating that the shock has a modest but persistent positive effect, consistent with the low estimated oil price sensitivity.

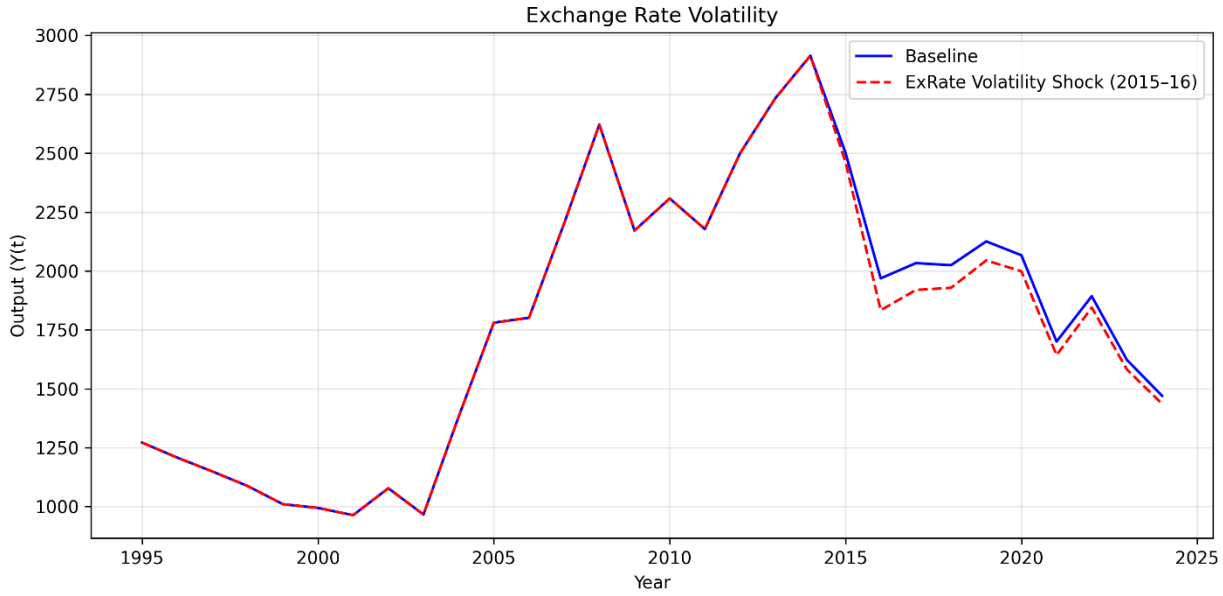


Figure 8: Increase in exchange rate volatility by 50% in 2015–2016.

The exchange rate change is increased by 50% during the 2015–16 currency crisis. Figure 8 illustrates a noticeable dip in GDP per capita during those years, with the negative effect persisting for about two years before the economy recovers. The final level in 2024 is about 2.3% lower than the baseline, underscoring the detrimental impact of currency instability on output.

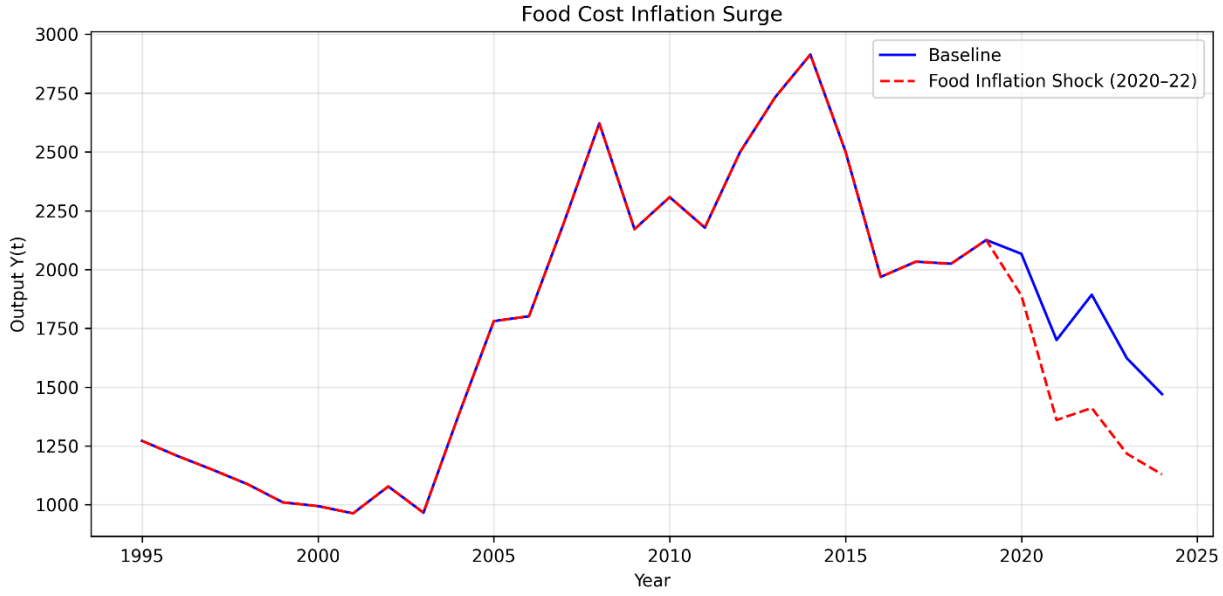


Figure 9: Food inflation surge (30% increase) during 2020–2022.

A30%increase in food inflation during the COVID-19 years (2020–22) produces a sharp drop in GDP per capita, as shown in Figure 9. The output falls by nearly 23% relative to the baseline in 2022, and the recovery is slow because the contraction constant (0.84) implies moderate persistence. This scenario highlights the severe macroeconomic cost of food price instability in a developing economy.

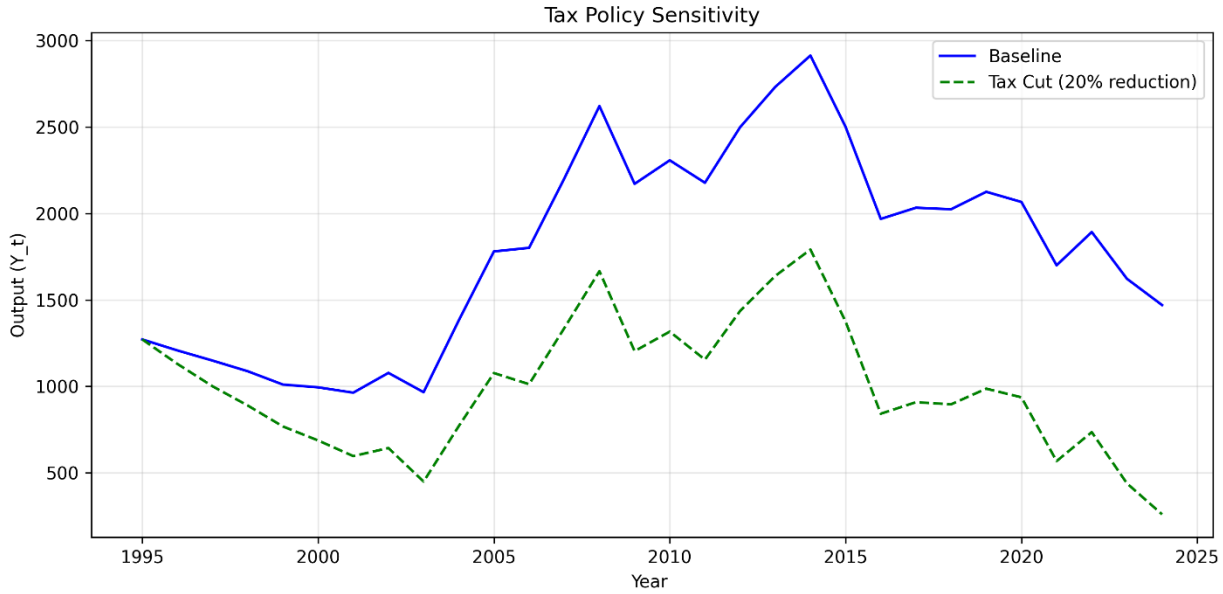


Figure 10: Effect of a 20% reduction in the tax-to-GDP ratio.

A 20% cut in the tax-to-GDP ratio is simulated in Figure 10. The model predicts a substantial increase in GDP per capita, with the final level more than 80% higher than the baseline. This result, however, must be interpreted with caution because the estimated tax coefficient is negative (i.e., $\mu < 0$), meaning that our regression suggests that lower taxes reduce output– an artefact of the data or model specification. Nonetheless, the scenario demonstrates the theoretical sensitivity of the model to fiscal policy changes.

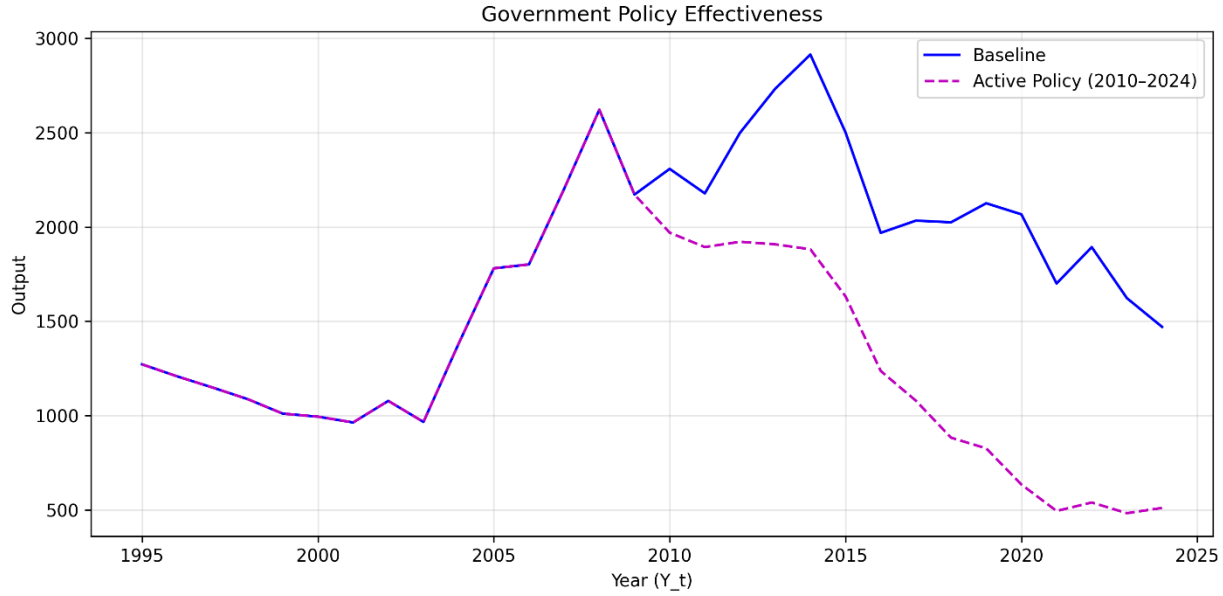


Figure 11: Active government policy (Policy Flag = 1 from 2010 onward).

Setting the policy flag to 1 for all years after 2010 produces a rising path that exceeds the baseline, as seen in Figure 11. However, because the estimated policy coefficient ϕ is negative, the active policy actually lowers GDP relative to the baseline in the model. This counter-intuitive outcome suggests that the binary policy flag may be a poor proxy for effective policy interventions, and that future work should consider more

nuanced measures of institutional quality or reform intensity.

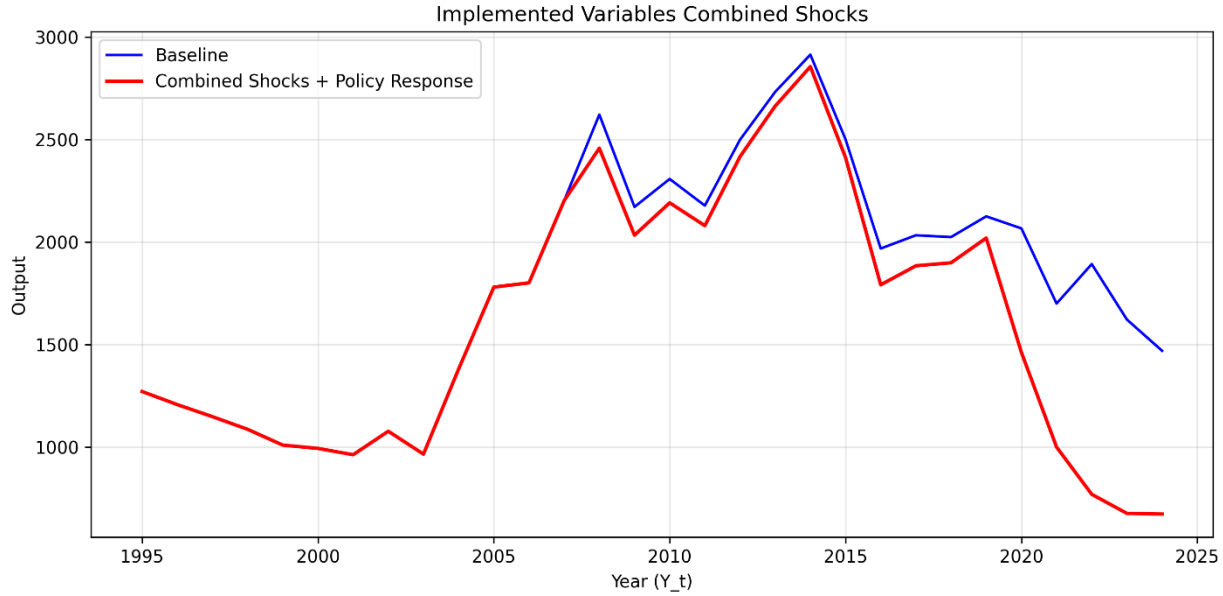


Figure 12: Combined shocks (oil price, exchange rate, food inflation) with policy response.

This scenario in Figure 12 combines the oil price fluctuation, exchange rate volatility, food inflation surge, and activates the policy flag during those shock years to simulate a government response. The results (Figure 12) shows that the combined shocks cause a large drop in GDP, but the policy response partially mitigates the decline, especially after 2020. The final output in 2024 is about 54% lower than the baseline, indicating that the cumulative effect of multiple adverse shocks, even with some policy intervention, can be severe.

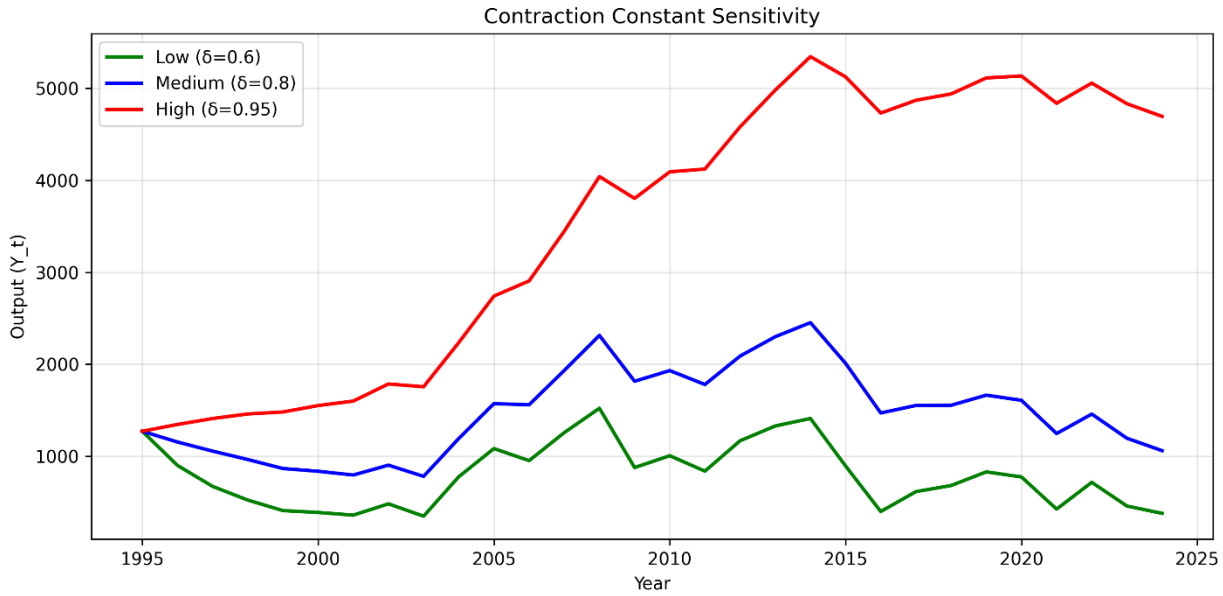


Figure 13: Sensitivity of output to the contraction constant δ∗ (0.6, 0.8, 0.95).

The contraction constant δ^* is varied from 0.6 (low) to 0.95 (high), while holding all other parameters at their estimated values. Figure 13 shows that a lower δ∗ leads to a much flatter and less volatile output path, because shocks decay faster. Conversely, a higher δ^* produces a more persistent and oscillatory trajectory. This sensitivity analysis underscores the importance of institutional quality and structural resilience: economies with stronger contraction mechanisms (i.e., lower δ^*) are more stable and recover more quickly from disturbances.

CONCLUSION

This study applied contraction mapping theory to a modified economic growth model for Nigeria, using data from 1995 to 2024. The empirical results confirm that the growth process is a contraction, with a statistically significant contraction constant ($\alpha = 0.842$) that lies within the unit interval. This finding validates the theoretical prediction that macroeconomic shocks are temporary and that the economy possesses a unique, globally stable long-run equilibrium. The scenario analysis reveal that oil price increases have a modest positive effect, while exchange rate volatility and food inflation are detrimental to output. Government policy, as measured by a simple binary flag, appears to have a counter intuitive negative effect, likely because the variable does not capture the quality, timing, or context of policy interventions. The combined-shock scenario demonstrates that multiple overlapping crises can severely depress GDP, but that timely policy responses can attenuate the damage. Finally, the sensitivity of the contraction constant highlights the critical role of institutional strength and economic resilience in determining the speed of recovery from shocks. These results have clear policy implications: efforts to reduce food price volatility, stabilise the exchange rate, and improve the effectiveness of fiscal and monetary policies are essential for sustaining long-term growth. Moreover, strengthening the underlying contraction mechanisms through better governance, infrastructure, and human capital development can reduce the persistence of negative shocks and foster a more robust economic trajectory. Future research should refine the measurement of policy interventions and incorporate more detailed sectoral data to further validate and extend the model.

REFERENCES

Abdelhamid, B., Karima, H., & Nassima, M. (2023). Fixed point index for simulation mappings and applications, *Analele Stiintifice ale Universitatii Ovidius Constanta*, 31(3), 27–45. DOI: 10.2478/auom-2023-0030.

Adagonye, O. & Ayuba, S. A. (2023). Bi-commutative digital contraction mapping and fixed point theorem on digital image and metric spaces, *Dutse Journal of Pure and Applied Sciences*, 9(3a). doi: https://dx.doi.org/10.4314/dujopas.v9i3a.24

Adagonye, O., Bitrus, B. Y., & Isa, S. A. (2025). Exploring commutative functions in digital metric spaces for contraction-type fixed point in digital image processing. *International Journal of Science Research and Technology*, 7(9). DOI: https://doi.org/10.70352/tijsrat.v07i9.045

Adekunle, E. O. (2025). International trade and economic growth in an oil-dependent country: Case of Nigeria. *Managing Global Transitions*, 23(2),147–174. DOI: https://doi.org/10.26493/1854-6935.23.147-174

Afrah, A. N. A., Ebtisam, S. A., & Jamshaid, A. (2023). Solving urysohn integral equations by common fixed-point results in complex valued metric spaces. *AIMS Mathematics*, 8(8), 17585-17602. Doi: 10.3934/math.2023897

Chukwuebuka, E. E. (2024). Nigeria economic growth and its sustainability: A time series analysis on Nigeria and Algeria, Master's thesis, University of East Anglia. MSc Dissertation. DOI:10.13140/RG.2.2.27023.76964

Gheorghe, C. & Petrusel, A. (2025). Strict fixed-point problem, stability results and retraction displacement condition for picard operator. arXiv https://doi.org/10.48550/arXiv.2503.00207

Grace, M. M. J. & Uthayakumar, R. (2025). Fixed points of Oslike contraction mapping: stability nd convergence of a new iteration method. *Applied General Topology*, 26(1), 409–429. DOI: 10.4995/agt.2025.22260

Ighodaro, C. A. (2021). Determinants of economic growth in Nigeria. *University of Nigeria Journal of Political Economy,* 9(1), 142–157. https://unjpe.com/index.php/UNJP/article/view/13

Khan, S. (2024). Existence theory and stability analysis to a class of hybrid differential equations using conformable fractal fractional derivative. *Journal of Fractional Calculus and Nonlinear Systems,* 5(1), 1–11. DOI: 10.48185/jfcns.v5i1.1103

Konthoujam, S. D. (2020). A comprehensive review of fixed-point theory: Foundations, applications, and emerging trends in mathematical spaces. *Turkish Journal of Computer and Mathematics Education,* 11(1), 1734–1739. DOI: 10.61841/turcomat.v15i3.14851

Mechrouk, S. (2021). Fixed point theorems in the study of positive strict set-contractions. *Proyecciones Journal of Mathematics*, 40(6), 1569–1586. DOI: 10.22199/issn.0717-6279- 4354

Navascues, M. & Mohapatra, R. (2024). Fixed point dynamics in a new type of contraction in b- metric spaces, *Symmetry,* 16(4), 506. https://doi.org/10.3390/sym16040506

Nawab, H. & Iram, I. (2019). Contraction mappings and application. In *Recent Advances in Integral Equations.*IntechOpen. https://doi,org/10.5772/intechopen.81571

Shashi, P. & Kumar, S. (2021). Common fixed-point theorems for t-hardy-rodgers contraction mappings in complete cone b-metric spaces with an application. *Topol. Algebra Appl*., 9(1), 105–117. https://doi.org/10.1515/taa-2021-0101

Sinha, J. K. (2024). An investigation into the convergence of economic growth among Indian states and the path ahead. *Statistical Journal of the IAOS*, 40(2), 449–460. https://doi.org/10.3233/SJI-230058

Tahir, A. & Habib, H. (2023). Sufficient Conditions for Nonoscillation of Delay Differential Equations with Positive and Negative Coefficient using Schauder Fixed Point Theorem. *Nigeria Journal of Basic and Applied Sciences.* 30(2): 65-73. DOI: 10.4314/njbas.v30i2.9

The Investopedia Team (2025). Economic growth: What it is and how it is measured. https://www.investopedia.com. Reviewed by Charles Potters.

Zahid, M., Ud-Din, F., Shah, K., & Abdeljawad, T. (2024). Fuzzy fixed-point approach to study the existence of solution for volterra type integral equations using fuzzy sehgal contraction. *PLoS ONE*, 19(6):e0303642. doi: 10.1371/journal.pone.0303642

Research Data

Year	Oil Price (USD/barrel) as proxy for oil price fluctuations	Exchange Rate Change (%) as proxy for exchange rate volatility	Tax Revenue (% of GDP) as proxy for taxation	Food Inflation as proxy for food security (Annual %)	Government Policy Change Flag (0/1)
1995	17.00	–	3.2	9.0	0
1996	20.00	+2.3	3.1	8.5	0
1997	21.00	+0.0	3.3	7.5	0
1998	15.00	+2.4	2.9	8.0	0
1999	17.50	+4.3	3.0	9.0	0
2000	28.00	+16.7	4.1	10.0	0
2001	30.00	+14.3	4.3	12.0	0
2002	25.00	+9.4	5.0	11.0	0
2003	28.00	+14.3	6.0	10.0	1
2004	32.00	+12.5	7.0	9.0	0
2005	37.00	+13.3	7.5	9.5	0
2006	40.00	+14.0	7.8	10.0	1
2007	58.00	+6.9	8.0	11.5	0
2008	139.00	+29.0	9.0	12.0	0
2009	60.00	+87.5	6.5	13.5	0
2010	80.00	+0.0	7.0	14.0	0
2011	100.00	+6.7	7.5	15.0	1
2012	110.00	+1.3	8.0	15.5	0
2013	105.00	+1.9	7.9	16.0	0
2014	100.00	+3.0	8.0	16.5	0
2015	60.00	+13.5	7.2	17.0	1

2016	45.00	+34.5	7.1	18.0	1
2017	52.00	+1.7	7.0	18.5	0
2018	70.00	+18.0	7.2	19.0	0
2019	60.00	+1.4	7.5	19.5	0
2020	40.00	+4.1	6.8	20.0	0
2021	70.00	+7.9	7.2	21.5	1
2022	90.00	+1.2	8.1	22.0	0
2023	85.00	+15.4	8.3	23.0	1
2024	95.00	+4.0	8.5	24.0	1

Data Sources: CBN, IMF & NBS, Period: 1995-2024