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A Generalized Extension of Ampadu-Unit Teissier Distribution: Derivation and Structural Properties

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ABSTRACT

In this paper, a new modified probability distribution called A Generalized Extension of Ampadu-Unit Teissier distribution (AUTD) which extends the baseline Unit Teissier (UTD) distribution was proposed. Nevertheless, the new model distribution is capable of handling asymmetric data sets. Some statistical properties such as moments, moment generating function, quantile function, and order statistics were derived and presented theoretically. Also, we proceed to test the validity of the new constructed probability distribution. A simulation was conducted to determine the capability and efficiency of the estimated parameters value by increasing the sample size. The results show that when simulated data are being used the new probability distribution provides a better fit than the competitive models.

Keywords:
Ampadu,
Teissier distribution,
Asymmetry

INTRODUCTION

In recent years, the development of continuous probability distributions for bounded and survival data has gained considerable attention in statistical modeling. A common challenge in practical applications is the effective handling of asymmetry, which often limits the flexibility of traditional distributions. To address this issue, Halidu et al. (2025) introduced a novel modified distribution defined on the unit interval, termed the Lomax-Unit Teissier Distribution (LxUTD). As an extension of the existing Unit Teissier Distribution (UTD), the LxUTD is specifically designed to manage asymmetric datasets. The authors provided a comprehensive theoretical foundation by deriving key statistical properties, including moments, the moment generating function, Renyi entropy, quantile function, and order statistics. The Unit-Teissier distribution is a continuous probability distribution defined on the unit interval (0,1) and is essential for modeling proportions, rates, and other bounded measurements prevalent in fields such as finance, medicine, and engineering. The Unit-Teissier Distribution (UTD), introduced as a bounded variant of the classical Teissier distribution, has emerged as a useful model for such data (Krishna *et al.*, 2022). However, its single-parameter structure imposes limitations, rendering it insufficiently flexible to capture complex real-world data features like asymmetry and heavy tails. This inflexibility curtails its practical application and reliability for diverse datasets. The Ampadu-G family of distributions, known for its shape and scale adaptability, offers a powerful framework for generalizing baseline models (Ampadu, 2019). This study addresses the rigidity of the UTD by integrating it within the Ampadu-G family. The primary aim is to develop a more flexible two-parameter model, the Ampadu-Unit Teissier Distribution (AUTD). The specific objectives are to derive its core distribution functions and essential structural properties, establish its validity as a proper probability density, and outline the method for parameter estimation via Maximum Likelihood.

MATERIALS AND METHODS

The Unit Teissier distribution

Krishna et al. (2022) proposed the one-parameter called unit Teissier distribution. The UTD is the extension of Teissier distribution by the French biologist Georges Teissier (1934). A random variable X is said to follow the UTD with one scale parameter α > 0, if its CDF is of the following form:

$$G(x;\alpha)=x^{-\alpha}e^{-x^{-\alpha}+1}, \quad x\in[0,1] \tag{1}$$

and probability density function (PDF) is given by:

$$g(x;\alpha)=\alpha(x^{-\alpha}-1)x^{-\alpha-1}e^{-x^{-\alpha}+1} \tag{2}$$

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From equations (1) and (2) above, the survival function S(x;α) and hazard rate function h(x;α) of UTD are obtained as follows:

$$S(x;\alpha)=1-x^{-\alpha}\exp(-x^{-\alpha}+1) \quad (3)$$

$$h(x;\alpha)=\frac{\alpha(x^{-\alpha}-1)x^{-(\alpha+1)}\exp(-x^{-\alpha}+1)}{1-x^{-\alpha}\exp(-x^{-\alpha}+1)} \quad (4)$$

Ampadu-G Family

Ampadu (2019) proposed a family of distribution namely Ampadu-G family of distributions based on the T–X(W) family framework.

The CDF and PDF of the proposed Ampadu-G family are given in (5) and (6) respectively:

$$F\left(x;\lambda,\zeta\right)=1-e^{-\lambda G(x;\zeta)^2}\left(1-e^{-\lambda}\right)^{-1} \quad (5)$$

And

$$f\left(x;\lambda,\zeta\right)=\frac{2\lambda g(x)G(x)e^{-\lambda G(x;\zeta)^2}}{1-e^{-\lambda}} \quad (6)$$

Where λ > 0 is the scale parameter and $\zeta > 0$ is the vector of parameters of the baseline CDF. From (5) and (6), the survival function S(x; λ) and hazard rate function h(x; λ) of the Ampadu-G family distributions are obtained as follows:

$$S\left(x;\lambda\right)=1-\left(1-\exp(-\lambda G(x;\zeta)^2)\left(1-\exp(-\lambda)\right)^{-1}\right) \quad (7)$$

$$h\left(x;\lambda\right)=\frac{2\lambda g(x)G(x)\exp\left(-\lambda G(x;\zeta)^2\right)}{1-\exp\left(-\lambda G(x;\zeta)^2\right)} \quad (8)$$

The New Modified Distribution

Generalized Extension of Ampadu-Unit Teissier Distribution

To obtain the CDF and PDF of the AUTD distribution, let the CDF and PDF of the Unit-Teissier distribution given in (1) and (2) are substituted into (5) and (6) respectively.

Then, the CDF of AUT distribution is obtained as in (9).

$$F\left(x;\lambda,\alpha\right)=1-\exp\left(-\lambda(x^{-\alpha}e^{-x^{-\alpha}+1})^2\right)\left(1-\exp\left(-\lambda\right)\right)^{-1} \quad (9)$$

And,

The corresponding PDF is given by:

$$f(x;\lambda,\alpha)=$$

$$\frac{2\lambda\alpha(x^{-\alpha}-1)x^{-2\alpha-1}}{1-\exp(-\lambda)} \quad (10)$$

$$\exp\left(-2x^{-\alpha}+2\right)\exp\left(-\lambda(x^{-\alpha}e^{-x^{-\alpha}+1})^2\right)$$

Where x >0, λ > 0 is the scale parameter and α > 0 is the shape parameter, respectively.

Relationship between CDF and PDF of the Modified Distribution

To prove that the derivative of the proposed CDF of the AUTD is equal to the PDF, and must satisfy that:

$$\frac{dF(x)}{dx}=f(x) \quad (11)$$

$$\frac{d}{dx}\left(\frac{1-e^{-\lambda(x^{-\alpha}e^{-x^{-\alpha}+1})^2}}{1-e^{-\lambda}}\right)=f(x) \quad (12)$$

Let

$$y=\left(1-e^{-\lambda(x^{-\alpha}e^{-x^{-\alpha}+1})^2}\right)\left(1-e^{-\lambda}\right)^{-1}, and\ u=x^{-\alpha}e^{-x^{-\alpha}+1}$$

Then,

$$y=\left(1-e^{-\lambda u^2}\right)\left(1-e^{-\lambda}\right)^{-1} \quad (13)$$

$$\frac{dy}{du}=2u\lambda e^{-\lambda u^2}(1-e^{-\lambda})^{-1},$$

$$\frac{du}{dx}=x^{-\alpha}\alpha x^{-\alpha-1}e^{-x^{-\alpha}+1}+e^{-x^{-\alpha}+1}(-\alpha x^{-\alpha-1})$$

Therefore,

$$\frac{dF(x)}{dx}=\frac{dy}{du}\cdot\frac{du}{dx} \quad (14)$$

$$\frac{dF(x)}{dx}=2\lambda\alpha(x^{-\alpha}-1)ux^{-\alpha-1}e^{-x^{-\alpha}+1}e^{-\lambda u^2}(1-e^{-\lambda})^{-1} \quad (15)$$

$$f(x;\lambda,\alpha)=\frac{2\lambda\alpha(x^{-\alpha}-1)x^{-\alpha-1}e^{-x^{-\alpha}+1}x^{-\alpha}e^{-x^{-\alpha}+1}e^{-\lambda u^2}}{1-e^{-\lambda}} \quad (16)$$

And hence (16) proved.

Model Validation of the Generalized Extension of Ampadu-Unit Teissier Distribution

To prove that the proposed probability density function (PDF) of the new AUTD is valid, it must satisfy that;

$$\int_0^1 f(x;\lambda,\alpha)dx=1 \quad (17)$$

$$\int_0^1 \frac{2\lambda\alpha(x^{-\alpha}-1)x^{-2\alpha-1}}{1-\exp(-\lambda)}\exp\left(-2x^{-\alpha}+2\right)\exp\left(-\lambda(x^{-\alpha}e^{-x^{-\alpha}+1})^2\right)dx=1 \quad (18)$$

Let

$$u=x^{-\alpha},\ \frac{du}{dx}=-\alpha x^{-\alpha-1}\Rightarrow dx=\frac{du}{-\alpha x^{-\alpha-1}}$$

$$\frac{2\lambda}{1-e^{-\lambda}}\int_0^1(1-u)e^{-u+1}ue^{-u+1}e^{-\lambda(ue^{-u+1})^2}\,du \quad (19)$$

Let

$$m=ue^{-u+1},\frac{dm}{du}=(1-u)e^{-u+1}\Rightarrow du=\frac{dm}{(1-u)e^{-u+1}}$$

$$\frac{2\lambda}{1-e^{-\lambda}}\int_0^1 me^{-\lambda m^2}\,dm \qquad (20)$$

Let

$$t=-\lambda m^2,\frac{dt}{dm}=-2\lambda m \Rightarrow dm=\frac{dt}{-2\lambda m}$$

$$\frac{1}{1-e^{-\lambda}}\int_{-\lambda}^{0} e^{t}\,dt \qquad (21)$$

$$\int_0^1 f(x;\lambda,\alpha)=\frac{1}{1-e^{-\lambda}}1-e^{-\lambda}=1 \qquad (22)$$

Hence, the model in equation (10) is a valid probability density function for Generalized Extension of Ampadu-Unit Teissier Distribution.

Survival Function and Hazard Rate Function for Generalized Extension of Ampadu-Unit Teissier Distribution

To obtain the survival function we substitute (1) into (7), it will give us (23).

$$S(x;\alpha,\lambda)=\exp\left(-\lambda(x^{-\alpha}e^{-x^{-\alpha}+1})^2\right)-\exp(-\lambda)\left(1-\exp(-\lambda)\right)^{-1} \qquad (23)$$

Also, to obtain the hazard rate function is to substitute (1) and (2) into (8) respectively:

$$h(x;\alpha,\lambda)=\frac{2\alpha\lambda(x^{-\alpha}-1)x^{-2\alpha-1}\exp\left(-2\left(x^{-\alpha}-1\right)\right)}{1-\exp\left(-\lambda\left(1-\left(x^{-\alpha}e^{-x^{-\alpha}+1}\right)^2\right)\right)} \qquad (24)$$

Where x >0 and λ > 0 is the scale parameter and α > 0 is the shape parameter respectively.

Linear Representation for the Density Function of AUT distribution

From (10) above, consider:

$$f(x;\lambda,\alpha)=\frac{2\lambda\alpha(x^{-\alpha}-1)x^{-2\alpha-1}}{1-\exp(-\lambda)}\exp\left(-2x^{-\alpha}+2\right)\exp\left(-\lambda(x^{-\alpha}e^{-x^{-\alpha}+1})^2\right) \qquad (25)$$

$$Let\ w=(x^{-\alpha}-1)\Rightarrow x=(w+1)^{-\frac{1}{\alpha}},\frac{dw}{dx}=-\alpha x^{-\alpha-1}$$

$$g(x)=f(x)\left|\frac{dx}{dw}\right| \qquad (26)$$

$$g(x)=\frac{2\lambda}{1-e^{-\lambda}}w(w+1)e^{-2w}e^{-\lambda(w+1)^2e^{-2w}} \qquad (27)$$

$$f(x)=\frac{2\lambda}{1-e^{-\lambda}}w(w+1)e^{-2w}e^{-\lambda(w+1)^2e^{-2w}},w\geq 0 \qquad (28)$$

Moments for Generalized Extension Ampadu-Unit Teissier Distribution

Let X be a random variable that have a Generalized Extension Ampadu-Unit Teissier distribution, then the rth moment about the origin is given as:

$$M_x=E(X^r)=\int_0^\infty X^r f(x)dx \qquad (29)$$

$$E\left(X^r\right)=\int_0^\infty X^r\frac{2\lambda}{1-e^{-\lambda}}w(w+1)e^{-2w}e^{-\lambda(w+1)^2e^{-2w}}dx \qquad (30)$$

$$Let\ w=(x^{-\alpha}-1)\Rightarrow x=(w+1)^{-\frac{1}{\alpha}}\Rightarrow X^r=(w+1)^{-\frac{r}{\alpha}}$$

$$E\left(X^r\right)=\frac{2\lambda}{1-e^{-\lambda}}\int_0^\infty\begin{pmatrix}(w+1)^{-\frac{r}{\alpha}}w(w+1)e^{-2w}\times\\ e^{-\lambda(w+1)^2e^{-2w}}\end{pmatrix}dw \qquad (31)$$

$$e^{-\lambda(w+1)^2e^{-2w}}=\sum_{k=0}^\infty\frac{\left(-\lambda(w+1)^2e^{-2w}\right)^k}{k!} \qquad (32)$$

$$e^{-\lambda(w+1)^2e^{-2w}}=\sum_{k=0}^\infty\frac{(-\lambda)^k}{k!}(w+1)^{2k}e^{-2kw} \qquad (33)$$

$$E\left(X^r\right)=\frac{2\lambda}{1-e^{-\lambda}}\sum_{k=0}^\infty\frac{(-\lambda)^k}{k!}\int_0^\infty\begin{pmatrix}w(w+1)^{2k+1-\frac{r}{\alpha}}\times\\ e^{-2(k+1)w}\end{pmatrix}dw \qquad (34)$$

$$E\left(X^r\right)=\frac{2\lambda}{1-e^{-\lambda}}\sum_{k=0}^\infty\frac{(-\lambda)^k}{k!}.a_{k,r} \qquad (35)$$

$$where\ a_{k,r}=\int_0^\infty w(w+1)^{2k+1-\frac{r}{\alpha}}e^{-2(k+1)w}dw$$

Therefore, the moments (35) is obtained by applying the Tricomi (confluent hyper geometric) U-function. Using the standard integral representation given as:

$$\int_0^\infty x^{b-1}(1+x)^{a-b-1}e^{-cx}dx=\Gamma(b)U(b,a,c) \qquad (36)$$

From (35) above we find the corresponding of a, b, and c.

We have:

$$a=2k+4-\frac{r}{\alpha},b=2,and\ c=2(k+1)$$

Thus:

$$a_{k,r}=U\left(2,2k+4-\frac{r}{\alpha},2(k+1)\right)$$

$$E\left(X^r\right)=\frac{2\lambda}{1-e^{-\lambda}}\sum_{k=0}^\infty\frac{(-\lambda)^k}{k!}U\left(2,2k+4-\frac{r}{\alpha},2(k+1)\right) \qquad (37)$$

Moment Generating Function for Generalized Extension Ampadu-Unit Teissier distribution

Let X be a random variable that have a Generalized Extension Ampadu-Unit Teissier distribution, then the rth moment about the origin, E(e^{tx}) is given as:

$$M_X(t)=E\left(e^{tx}\right)=\int_0^\infty e^{tx}f\left(x\right)dx \tag{38}$$

$$E\left(e^{tx}\right)=\int_0^\infty e^{tx}\frac{2\lambda}{1-e^{-\lambda}}w(w+1)e^{-2w}e^{-\lambda(w+1)^2e^{-2w}}dx \tag{39}$$

Let

$$x=(w+1)^{-\frac{1}{\alpha}}$$

$$E\left(e^{Xt}\right)=\frac{2\lambda}{1-e^{-\lambda}}\int_0^\infty\left(e^{t(w+1)^{-\frac{1}{\alpha}}}w(w+1)e^{-2w}e^{-\lambda(w+1)^2e^{-2w}}\right)dw \tag{40}$$

Characteristics Function for Generalized Extension Ampadu-Unit Teissier distribution

Let X be a random variable that have a Generalized Extension Ampadu-Unit Teissier distribution, then the rth moment about the origin, E(e^{itx}) is given as:

$$\phi_X(t)=E\left(e^{itX}\right)=\int_0^\infty e^{itx}f\left(x\right)dx \tag{41}$$

$$E\left(e^{itX}\right)=\int_0^\infty e^{itX}\frac{2\lambda}{1-e^{-\lambda}}w(w+1)e^{-2w}e^{-\lambda(w+1)^2e^{-2w}}dx \tag{42}$$

Let

$$x=(w+1)^{-\frac{1}{\alpha}}$$

$$E\left(e^{itX}\right)=\frac{2\lambda}{1-e^{-\lambda}}\int_0^\infty\left(e^{it(w+1)^{-\frac{1}{\alpha}}}w(w+1)e^{-2w}e^{-\lambda(w+1)^2e^{-2w}}\right)dw \tag{43}$$

Quantile Function for Generalized Extension Ampadu-Unit Teissier distribution

Let X be a random variable that has the CDF given in (9). The quantile function Q(u) can be derived as follows:

$$F(x)=\frac{1-e^{-\lambda(x^{-\alpha}e^{-x^{-\alpha}+1})^2}}{1-e^{-\lambda}} \tag{44}$$

Let

$$F(x)=u \tag{45}$$

$$u=\frac{1-e^{-\lambda(x^{-\alpha}e^{-x^{-\alpha}+1})^2}}{1-e^{-\lambda}} \tag{46}$$

By separating variable x in (46) and taking the natural logarithm we have:

$$x^{-\alpha}e^{-x^{-\alpha}+1}=\sqrt{-\frac{1}{\lambda}In\left(1-u(1-e^{-\lambda})\right)} \tag{47}$$

Let

$$c=\sqrt{-\frac{1}{\lambda}In\left(1-u(1-e^{-\lambda})\right)}\Rightarrow x^{-\alpha}e^{-x^{-\alpha}}=ce^{-1},z=x^{-\alpha}$$

$$ze^{-z}=ce^{-1} \tag{48}$$

Therefore, the quantile function (48) is obtained by applying Lambert W function

$$Q(u)=\left(-W\left(-\sqrt{-\frac{1}{\lambda}In\left(1-u(1-e^{-\lambda})\right)}.e^{-1}\right)\right)^{-\frac{1}{\alpha}} \tag{49}$$

Order Statistics for Generalized Extension Ampadu-Unit Teissier distribution

For a random sample X$_1$, X$_2$,……….., X$_n$ of independent and identically distributed (i.i.d.) random variables from the AUTD, the order statistics can be obtained by:

$$f_{k:n}(x;\alpha,\lambda)=\frac{n!}{(k-1)!(n-k)!}\left[F(x;\alpha,\lambda)\right]^{k-1}\left[1-F(x;\alpha,\lambda)\right]^{n-1}f(x;\alpha,\lambda) \tag{50}$$

$$f_{k:n}(x;\alpha,\lambda)=\frac{n!}{(k-1)!(n-k)!}\left[\frac{1-e^{-\lambda\left(x^{-\alpha}e^{-x^{-\alpha}+1}\right)^2}}{1-e^{-\lambda}}\right]^{k-1}\left[\frac{e^{-\lambda\left(x^{-\alpha}e^{-x^{-\alpha}+1}\right)^2}-e^{-\lambda}}{1-e^{-\lambda}}\right]^{n-1} \tag{51}$$

$$\frac{2\alpha\lambda(x^{-\alpha}-1)e^{-2\left(x^{-\alpha}-1\right)}e^{-\lambda\left(x^{-\alpha}e^{-x^{-\alpha}+1}\right)^2}}{1-e^{-\lambda}}$$

The Parameter Estimation for Generalized Extension of Ampadu-Unit Teissier distribution

Let {x$_1$,x$_2$,……,x$_n$} be a random sample from the AUTD, the likelihood function is given as:

$$L(\alpha,\lambda)=\prod_{i=1}^{n}f(x_i;\alpha,\lambda) \tag{52}$$

And the log-likelihood is:

$$l(\alpha,\lambda)=\sum_{i=1}^{n}\log f(x_i;\alpha,\lambda) \tag{53}$$

$$l(\alpha,\lambda)=\sum_{i=0}^{n}\begin{bmatrix}\log 2+\log\alpha+\log\lambda+\\ \log\left(x_i^{-\alpha}-1\right)+(-2\alpha-1)\log x_i\\ -2\left(x_i^{-\alpha}-1\right)-\\ \lambda\left(x_i^{-\alpha}e^{-x_i^{-\alpha}+1}\right)^2-\log\left(1-e^{-\lambda}\right)\end{bmatrix} \tag{54}$$

$$\frac{\partial l}{\partial\alpha}=0\ and\ \frac{\partial l}{\partial\lambda}=0$$

$$\frac{\partial l}{\partial\alpha}=\frac{n}{\alpha}+\sum_{i=0}^{n}\begin{bmatrix}\dfrac{-x_i^{-\alpha}Inx_i}{x_i^{-\alpha}-1}-2Inx_i+2x_i^{-\alpha}Inx_i-2\lambda e^{2\left(-x_i^{-\alpha}+1\right)}.x_i^{-2\alpha}.\\ \left(x_i^{-\alpha}-1\right).Inx_i\end{bmatrix} \tag{55}$$

$$\frac{\partial l}{\partial\lambda}=\frac{n}{\lambda}-\frac{ne^{-\lambda}}{1-e^{-\lambda}}-\sum_{i=1}^{n}\left(x_i^{-\alpha}e^{-x_i^{-\alpha}+1}\right)^2 \tag{55}$$

The equation (54) and (55) will gives the estimate of the maximum likelihood estimate of the parameters.

RESULTS AND DISCUSSION

Plots of the CDF and PDF of Generalized Extension of Ampadu-Unit Teissier distribution

The plots of cumulative distribution function and probability density function of Generalized Extension of Ampadu-Unit Teissier distribution for varying parameter values are given in the figures below:

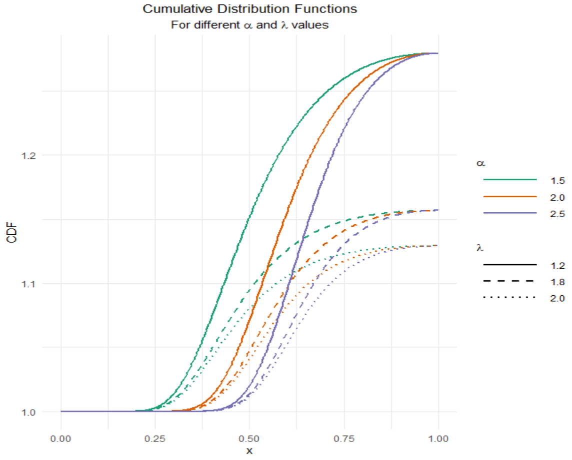

Figure 1: CDF plot of AUTD

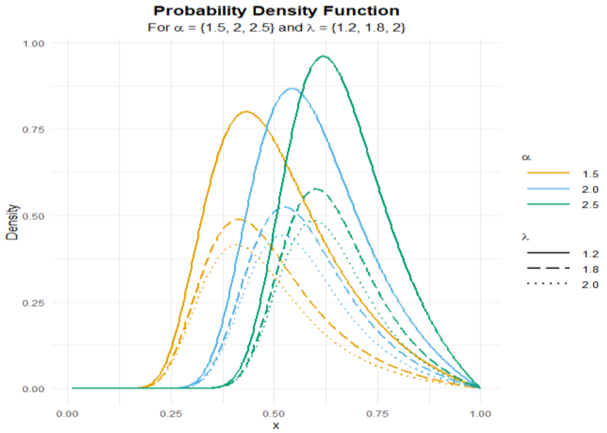

Figure 2 PDF plot of AUTD

Figure (1) shows the Cumulative Distribution Function of the AUTD for different values of the scale and shape parameters. Figure (2) displays the density function of AUTD for different values of the scale and shape parameters. Therefore, it exhibits both left and right skewed distribution and they display different level of kurtosis which shows that the flexibility of the distribution for modeling asymmetric datasets.

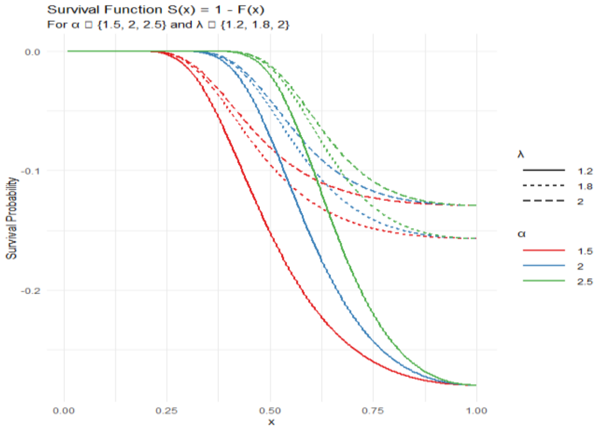

Figure 3: Survival plot of AUTD

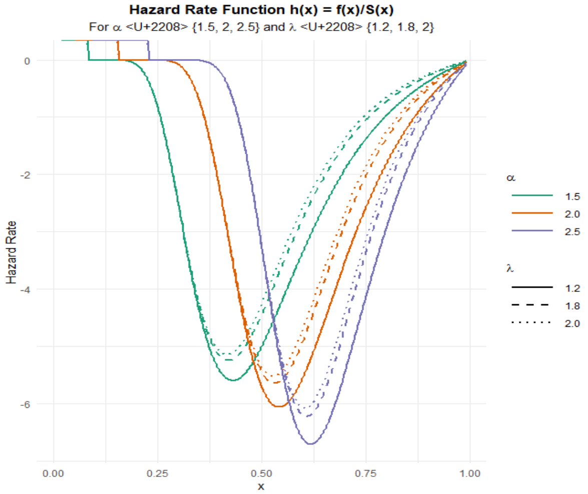

Figure 4: Hazard plot of AUTD

From figure (3) above of survival function shows varying shapes decreasing, constant and reversed-J shapes. Also, figure (4) of hazard rate plot show varying shapes like bathtub, reversed-J shape and nearly constant failure rate. The derivation successfully yielded the CDF and PDF of the novel AUTD. The relationship *dF(x)/dx=f(x)* was proven analytically, confirming internal consistency. Furthermore, the integration of the PDF over its support (0 to 1) was demonstrated to equal 1, establishing it as a valid probability density. The expressions for the survival and hazard functions provide tools for reliability and survival analysis. The general formula for the r^{th} non-central moment is given by:

$$E\left(X^{r}\right)=\frac{2\lambda}{1-e^{-\lambda}}\sum_{k=0}^{\infty}\frac{(-\lambda)^{k}}{k!}U\left(2,2k+4-\frac{r}{\alpha},2(k+1)\right), \quad (37)$$

Where *U(.)* is the Tricomi confluent hypergeometric function. This allows for the computation of the mean, variance, skewness, and kurtosis. The quantile function, crucial for generating random variates and calculating percentiles, is expressed as:

$$Q(u)=\left(-W\left(-\sqrt{-\frac{1}{\lambda}In\left(1-u(1-e^{-\lambda})\right)}.e^{-1}\right)\right)^{-\frac{1}{\alpha}} \quad (49)$$

Where *W* is the Lambert W function. The log-likelihood function for a random sample is presented, and the system of equations for obtaining Maximum Likelihood Estimations (MLEs) is provided. These derived properties collectively demonstrate that the introduction of the scale parameter λ via the Ampadu-G family has significantly enriched the mathematical framework of the baseline UTD, endowing the AUTD with greater shape flexibility to model complex data patterns on the unit interval.

CONCLUSION

This study has successfully developed and characterized the Ampadu-Unit Teissier Distribution (AUTD), a two-parameter generalization of the Unit-Teissier Distribution. The comprehensive derivation of its CDF, PDF, survival function, hazard function, moments, quantile function, and order statistics provides a solid theoretical foundation. The establishment of the maximum likelihood estimation framework ensures its practical applicability. The AUTD, with its enhanced flexibility due to the added parameter, represents a significant theoretical advancement over the standard UTD and is poised to be a more capable model for statistical analysis of bounded data exhibiting asymmetry or heavy tails.

Conflict of Interest

The authors declare there is no competing interest regarding publication of this article

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