



## Comparative Analysis of Conditional Error Distributions in Asymmetric GARCH modelling

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### ABSTRACT

This study compares the performance of GARCH-type models; GJR-GARCH (1,1), EGARCH (1,1) and APARCH (1,1) under three distinct conditional distributions: Exponentiated Student t-distribution (ESTD), Skewed Student-t distribution (SSTD), and Skewed Generalized Error Distribution (SGED) on Bitcoin data. The log returns of the Bitcoin in dollars were obtained from the daily closing prices of Bitcoin from 1<sup>st</sup> January, 2014 to 1<sup>st</sup> August, 2025. This time frame captures a range of market conditions, including periods of relative stability and significant shock. The data for this study was obtained from <https://finance.yahoo.com> website. Descriptive statistics reveal the typical financial stylized facts of heavy tails and skewness. Analysis on the data using the Augmented Dickey-Fuller (ADF) test showed stationarity and also the existence of significant ARCH effects via the ARCH-LM test. Among the asymmetric GARCH specifications estimated using Bitcoin return data, EGARCH (1,1) with exponentiated Student- t distribution (ESTD) emerged as the best-performing error distribution. The model consistently produced the lowest Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and least forecasting values using the metrics (RMSE and MAE) relative to competing conditional error distributions, skewed student -t distributions (SSTD) and Skewed generalized error distribution (SGED). These findings suggest that the ESTD distribution provides a superior characterization of the heavy tails and asymmetry inherent in Bitcoin return volatility dynamics. The diagnostic results indicate that all volatility models adequately captured serial correlation and ARCH effects, except the GJR-GARCH(1,1)-SGED model, which exhibited significant residual autocorrelation. Overall, the EGARCH and APARCH specifications demonstrated superior model adequacy and are considered suitable for volatility forecasting

### Keywords:

GARCH models,  
Volatility,  
Bitcoin,  
Error Distributions,  
ARCH Effect,

### INTRODUCTION

Volatility modeling remains a central topic in financial econometrics, particularly in the analysis of asset returns characterized by time-varying variance, volatility clustering, and leptokurtosis. Since the seminal work of Engle (1982), who introduced the Autoregressive Conditional Heteroskedasticity (ARCH) framework and its extension by Bollerslev (1986) through the Generalized ARCH (GARCH) model, there has been substantial progress in capturing stylized facts of financial time series. These models have proven useful in risk management, derivative pricing, and portfolio.

Accurate modeling of innovations is essential for trustworthy volatility projections and financial decision-making, as Tsay (2010) highlighted and the probabilistic behavior of the entire conditional variance process is also determined by the innovation structure, as Gouriou (1997) emphasized. However, empirical evidence consistently shows that financial return distributions exhibit asymmetry and heavy tails, features that are not adequately captured under the assumption of conditional normality. This limitation has motivated the adoption of alternative conditional error distributions, such as the Student- t distribution, generalized error distribution (GED), and skewed variants.

In particular, the Student - t distribution has gained prominence due to its ability to accommodate excess kurtosis (Bauwens & Laurent 2005). To further account for asymmetry in volatility dynamics, asymmetric GARCH-type models such as the GJR-GARCH model proposed by Brockwell & Davis (2002) and the Exponential GARCH (EGARCH) model introduced by Glosten et al. (1993) have been widely employed. These models capture leverage effects, where negative shocks tend to have a larger impact on volatility than positive shocks of equal magnitude. Despite these advancements, the choice of conditional error distribution remains a critical factor influencing model performance. Standard distributions may still be insufficient in capturing complex tail behavior and varying degrees of skewness observed in financial data. However, the review of relevant literatures shows that Autoregressive Conditional Heteroscedasticity model (ARCH model) and its various extensions are being applied for modelling volatility of financial time series data in order to captures the stylized facts incorporated in these series such as volatility clustering, persistence, heavy tail distribution, and leverage effects. But less attention has been given to the contribution of the error distribution assumptions while modelling volatility. According to Nelson (1991), the wrong use of an appropriate error distribution in volatility model for financial time series may cause misspecification in volatility model, leptokurtic and autocorrelation behavior of such series. Recent studies have increasingly emphasized the importance of flexible conditional distributions and advanced GARCH-type specifications in improving volatility forecasting performance. For instance, Deebom & Essi (2017) evaluate multiple GARCH models under diverse distributional assumptions, highlighting the sensitivity of volatility forecasts to error distribution choice. Similarly, Akgun & Gulay (2025) demonstrated the relevance of skewed heavy-tailed distributions within realized GARCH frameworks, while Ampountolas (2024) and related studies show that incorporating asymmetry and fat tails significantly enhances model accuracy in financial and commodity markets. Existing literature on cryptocurrency volatility modeling has extensively employed symmetric and asymmetric GARCH-family models to capture the stylized characteristics of Bitcoin returns. However, many studies concentrate primarily on model structure while paying comparatively less attention to the role of alternative conditional error distributions in improving volatility estimation and forecasting accuracy. Furthermore, the Exponentiated Student-t Distribution (ESTD) remains underexplored in the context of asymmetric GARCH modeling despite the pronounced leptokurtosis and skewness observed in Bitcoin returns. In addition, prior studies often evaluate model adequacy using only in-sample fit measures without jointly assessing forecasting performance and residual

diagnostics. This creates a gap in the empirical literature regarding the most suitable conditional error distribution for asymmetric volatility modeling of Bitcoin returns.

Therefore, this study proposes the use of an exponentiated Student's t distribution proposed by Okereke et al. (2026) within asymmetric GARCH frameworks and conducts a comparative analysis against conventional conditional error distributions. The exponentiated Student's t distribution provides an appealing extension, offering additional shape parameters to better model skewness and kurtosis simultaneously. The objective is to evaluate the impact of distributional assumptions on volatility estimation accuracy and model fit by incorporating greater flexibility in the error structure.

## MATERIALS AND METHODS

### Source of data

The data for this study is the daily closing price of Bitcoin which is a global financial market expressed in dollars. The Bitcoin data used covers 11- years from 1<sup>st</sup> January, 2014 to 1<sup>st</sup> August, 2025. This time frame captures a range of market conditions, including periods of relative stability and significant shock. The data for this study was obtained from <https://finance.yahoo.com>. The python statistical package was used in data analysis.

### Computation of daily return series.

The daily returns were calculated as the natural logarithm of the simple gross return using the formular:

$$r_t = \ln \left( \frac{BTC_t}{BTC_{t-1}} \right) \times 100, t = 2 \dots n \quad (1)$$

Where,  $BTC_t$  is the daily closing price at the present day and  $BTC_{t-1}$  is the daily closing price at the previous day.

### Normality of Return Series

To ascertain the normality of the return series of Bitcoin, the Jacque- Bera test was applied. The Jacque Bera test statistic is given as:

$$JcB = \frac{n}{6} \left[ sk^2 + \frac{(ku - 3)^2}{4} \right] \quad (2)$$

### Stationarity test for the daily return series

The stationarity of the daily returns Bitcoin series was tested using the Augmented Dickey Fuller (ADF).

$$\Delta BTC_t = \omega + \lambda t + \gamma BTC_{t-1} + \sum_{i=1}^p \delta_i \Delta BTC_{t-i} + \varepsilon_t \quad (3)$$

Where  $\Delta BTC_t$  is the first difference of BTC return,  $\omega$  is the constant term,  $\lambda t$  is the deterministic trend,  $\gamma$  is the coefficient used to test stationarity, p is the lag length and  $\varepsilon_t$  is the error term.

The hypothesis is given as:

$$H_0 : \gamma = 0$$

$$H_0 : \gamma < 0$$

The test statistic is

$$t = \frac{\hat{\gamma}}{SE(\hat{\gamma})} \tag{4}$$

Where,  $SE(\hat{\gamma})$  is the standard error of the least squares estimate of the  $\gamma$  coefficient from the regression model.

**ARCH effect test**

To test for ARCH effect, the Lagrangian multiplier test of Engle was used. The null hypothesis is given as:

$$H_0 : \omega_1 = \dots = \omega_p = 0 \text{ versus}$$

$$H_1 : \omega_i \neq 0 \text{ for some } i \in \{1, \dots, p\}$$

The test statistic

$$F = \frac{(SSR_0 - SSR_1) / m}{SSR_1 (T - 2m - 1)} \tag{5}$$

where  $SSR_1 = \sum_{t=m+1}^T a_t^2$ , where  $SSR_1$  is the sum of square

$$\text{error, } SSR_0 = \sum_{t=m+1}^T (\varepsilon_t^2 - \bar{\omega})^2.$$

The test statistic is assumed to follow  $\chi^2$  distribution and the decision rule is to reject

Ho: if  $F > \chi_m^2(\alpha)$  and T is the number of observations.

**Innovation distribution used in the study**

**Exponentiated Student-t Distribution (ESTD)**

Okereke et al (2026) derived the Exponentiated Student t distribution as:

$Z_t \sim \text{ESTD}(C, V)$ , its PDF is given by:

$$g(x) = \frac{1}{c} \left[ \frac{x \beta \left(\frac{v}{2}, \frac{1}{2}\right)}{2\sqrt{v} \beta \left(\frac{1}{2}, \frac{v}{2}\right)} \left(1 + \frac{x^2}{v}\right)^{-\frac{v+1}{2}} \right]^{c-1} \cdot \frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right) \sqrt{v} \Gamma\left(\frac{1}{2}\right)} \left(1 + \frac{x^2}{v}\right)^{-\frac{v+1}{2}} \tag{6}$$

Where  $C > 0$ ,  $V > 0$  and C is the shape (exponentiation) parameter while V is the degree of freedom.

and  $B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$  is a beta function.

And the CDF is given as:

$$G(x) = \left[ \frac{1}{2} + \frac{x \beta \left(\frac{v}{2}, \frac{1}{2}\right)}{2\sqrt{v} \beta \left(\frac{1}{2}, \frac{v}{2}\right)} \left(1 + \frac{x^2}{v}\right)^{-\frac{v+1}{2}} \right]^c \tag{7}$$

**Skewed Student t distribution**

The skewed Student -t distribution extends the standard t distribution by incorporating a parameter that allows for asymmetry in the data.

$$f(z_t, \mu, \sigma, \rho, \lambda) = \begin{cases} bc \left[ 1 + \frac{1}{\rho-2} \left( b \frac{z_t - \mu}{1-\lambda} \right)^2 \right]^{\frac{\rho+1}{2}} \cdot z_t < -\frac{a}{b} \\ bc \left[ 1 + \frac{1}{\rho-2} \left( b \frac{z_t - \mu}{1-\lambda} \right)^2 \right]^{\frac{\rho+1}{2}} \cdot z_t \geq -\frac{a}{b} \end{cases} \tag{8}$$

Where  $\rho$  is the shape parameter with  $2 < \rho < \infty$  and  $\lambda$  is the skewness parameters with

$-1 < \lambda < 1$ ,  $\mu$  and  $\sigma^2$  are the variance of the skewed student -t distribution.

$$a = 4\lambda c \left( \frac{\rho-2}{\rho-1} \right), b = 1 + 3\lambda^2 - a^2, c = \frac{\Gamma\left(\frac{\rho+1}{2}\right)}{\sqrt{\pi(\rho-2)} \Gamma\left(\frac{\rho}{2}\right)}$$

**Skewed Generalized Error Distribution**

**The Skewed Generalized Error Distribution (SGED)**

is an extension of the symmetric Generalized Error Distribution (GED) that incorporates asymmetry to better capture skewness commonly observed in financial return series. While the standard GED accommodates varying tail thickness through a shape parameter, it assumes symmetry about the mean, which is often unrealistic in empirical financial data.

$$f\left(z_t / \rho, \varepsilon, \theta, \delta\right) = \frac{\rho}{2\theta \Gamma\left(\frac{1}{\rho}\right)} \exp\left[ -\frac{|z_t - \delta|^\rho}{\left[1 + \text{sign}(z_t - \delta)\varepsilon\right]^\rho \theta^\rho} \right] \tag{9}$$

$$\theta > 0, -\infty < z_t, \rho > 0, -1 < \varepsilon < 1, -\infty < z_t < \infty$$

Where

$$\theta = \Gamma\left(\frac{1}{\rho}\right)^{0.5} \Gamma\left(\frac{3}{v}\right)^{-0.5} S(\varepsilon)^{-1} \tag{10}$$

$$\delta = 2\varepsilon S(\varepsilon)^{-1} \tag{11}$$

$$S(\varepsilon) = \sqrt{1 + 3\varepsilon^2 - 4A^2\varepsilon^2} \tag{12}$$

Where  $\rho > 0$  is the shape parameter,  $\mathcal{E}$  is the skewness parameter with  $-1 < \mathcal{E} < 1$

**Volatility Models used in the Study**

**Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Model:**

The GARCH model is written as:

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \tag{13}$$

Where

$\sigma_t^2$  is the conditional variance at time t,  $\omega$  is a constant,  $\alpha_i$  is the ARCH parameter capturing past squared shocks,  $\varepsilon_{t-i}^2$  is the previous shock and  $\beta_j$  is GARCH parameter capturing past conditional variances.

**Glosten Jaggannathan and Runkle Generalized Autoregressive Conditional Heteroscedasticity (GJR-GARCH) model**

The **GJR-GARCH model** specifies the conditional variance by explicitly allowing **asymmetric responses** of volatility to past shocks. The model is stated as:

$$\sigma_t^2 = \phi + \sum_{i=1}^p (\alpha_i \varepsilon_{t-i}^2 + \gamma_1 I_{t-i} \varepsilon_{t-i}^2) + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \tag{14}$$

Where

$\phi$  is constant term (long-run volatility)  $\alpha_i$  is ARCH term while  $\beta_j$  is the GARCH term,  $\gamma_1$  is the leverage term,  $\phi \geq 0$ ,  $\alpha_i$  and  $\beta_j \geq 0$  and  $\sigma_t$  is the volatility.

**Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH) Model**

The EGARCH model specifies the conditional variance in logarithmic form, ensuring that the variance is always positive without imposing non-negativity constraints on the parameters. The model is given as:

$$\ln(\sigma_t^2) = \phi + \sum_{i=1}^p \alpha_i \left[ \lambda \varepsilon_{t-i} + \gamma \left\{ |\varepsilon_{t-i}| - \sqrt{\frac{2}{\pi}} \right\} \right] + \sum_{j=1}^q \beta_j \ln(\sigma_{t-j}^2) \tag{15}$$

where,  $\phi$  is constant term,  $\alpha_i$  is ARCH term while  $\beta_j$  is the GARCH term and,  $\gamma$  is the leverage term and  $\sigma_t$  is the volatility.

**Asymmetric Power Autoregressive Conditional Heteroscedasticity [APARCH (p,q)] Model**

The APARCH model provides a flexible framework for modeling volatility by incorporating **asymmetry** and a **power transformation** of the conditional standard deviation. The model is stated as:

$$\sigma_t^2 = \phi + \sum_{j=1}^q \alpha_j \left( |\varepsilon_{t-j}| - \gamma_j \varepsilon_{t-j} \right)^\delta + \sum_{i=1}^p \beta_j (\sigma_{t-i})^\delta$$

where,

$\phi$  is constant term,  $\alpha_i$  is ARCH term while  $\beta_j$  is the GARCH term and  $\sigma_t$  is the volatility.

**Forecasting Specification**

The forecasting analysis was conducted using asymmetric GARCH-type models, namely the EGARCH(1,1), GJR-GARCH(1,1), and APARCH(1,1) models under different error distributions. The proposed Exponentiated Student- t distribution (ESTD) was compared with existing error distributions Skewed Student-t distribution (SSTD) and Skewed generalized error distribution (SGED) using Bitcoin return data. The dataset was divided into in-sample and out-of-sample periods. The in-sample observations were used for parameter estimation, while the out-of-sample observations were employed for volatility forecasting evaluation

**Model Selection and Forecasting Evaluation**

The fitness of the error distributions was compared using Log likelihood, Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC).

$$AIC = - \frac{2 \log(LL) + 2\beta}{n} \tag{16}$$

where,  $LL$  is the likelihood of the model and  $\beta$  is the number of parameters in the model and  $n$  is the number of observations.

$$BIC = - 2 \ln(L) + K + \ln(n) \tag{17}$$

Where  $L$  is the maximum value of the likelihood function for the model,  $K$  is the number of parameters in the model and  $n$  is the number of observations while the forecasting evaluation was done using), Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) as recommended by Franses (2016) and (Kahair et al.2017)

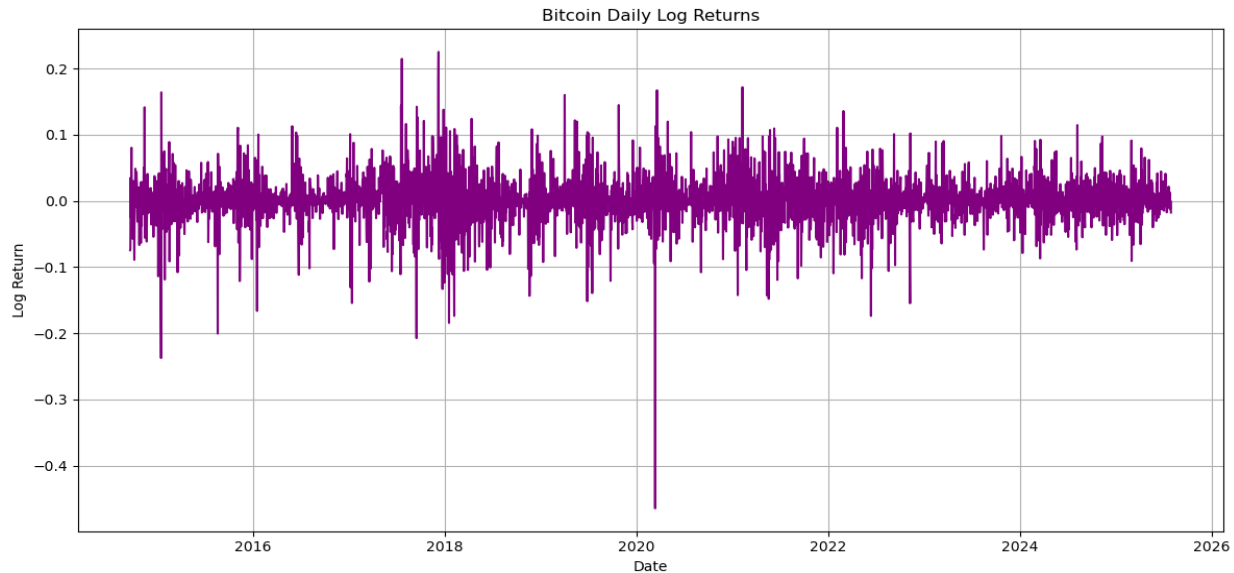
$$RMSE = \sqrt{\frac{1}{n-1} \sum_{t=2}^n (\sigma_t^2 - \hat{\sigma}_t^2)^2} \tag{18}$$

$$MAE = \frac{1}{n-1} \sum_{t=2}^n |\sigma_t^2 - \hat{\sigma}_t^2| \tag{19}$$

**RESULTS AND DISCUSSION**

**Application to data**

Fig. 1 presents the time plot of Bitcoin daily log returns. The plot fluctuates around a constant mean close to zero with no evident trend, indicating stationarity in mean.



**Fig 1: Time plot for the daily closing log returns for Bitcoin.**

From the Bitcoin daily log returns plot, the presence of volatility clustering suggests time-varying variance, implying conditional heteroscedasticity.

Table 1 presents the descriptive statistics for the bitcoin data.

| N    | Mean     | Maximum  | Minimum   | Std. Dev. | Skewness | Kurtosis | Jarque-Bera | p-value |
|------|----------|----------|-----------|-----------|----------|----------|-------------|---------|
| 3970 | 0.001394 | 0.225119 | -0.264730 | 0.033876  | -0.7162  | 14.5657  | 22466.5765  | 0.0000  |

The descriptive statistics of Bitcoin (BTC) daily log returns reveal that the mean return is 0.001394 and a

standard deviation of 0.033876 which reflects a relatively high level of volatility, which is typical of cryptocurrency markets. The distribution is **negatively skewed** (-0.7162), suggesting that extreme negative returns occur more frequently than extreme positive returns, indicating higher downside risk. The kurtosis value (14.5657) is **substantially greater than 3**, indicating a **leptokurtic distribution**. This implies that BTC returns exhibit **fat tails and a higher peak** than the normal distribution, meaning extreme values occur more often than would be expected under normality. The **Jarque-Bera statistic (22466.5765) with a p-value of 0.0000** strongly rejects the null hypothesis of normality. This confirms that BTC returns are **not normally distributed**.

**ADF Test for Stationarity and ARCH Effect Test**

Table 2: Augmented Dickey-Fuller (ADF) and ARCH effect result summary

| Series | ADF test       |          |            | ARCH effect       |         |                          |
|--------|----------------|----------|------------|-------------------|---------|--------------------------|
|        | Test Statistic | p-values | Comment    | ARCH test F-stat. | P-value | Comments                 |
| BTC    | - 19.1374      | 0.0000   | Stationary | 139.3089          | 0.0000  | Evidence of ARCH effects |

The ADF test statistic is **-19.1374** (p-value = **0.0000**) indicates that BTC returns are stationary since  $p < 0.05$ . The ARCH test result  $F = 139.3089$  (p-value = **0.0000**) confirms significant ARCH effects supporting the use of GARCH-type models.

**Estimation of the Parameters of the volatility models**

| Models           | $\mu$<br>(p-value) | $\phi$<br>(p-value)          | $\alpha$<br>(p-value)        | $\beta$<br>(p-value)        | $\gamma$<br>(p-value)       | Skew<br>(p-value)             | Shape<br>(p-value)          |                             |
|------------------|--------------------|------------------------------|------------------------------|-----------------------------|-----------------------------|-------------------------------|-----------------------------|-----------------------------|
| <b>GJR-GARCH</b> | <b>SSTD</b>        | <b>0.06284</b><br>(0.0036)   | <b>0.023028</b><br>(0.0553)  | <b>0.118300</b><br>(0.0000) | <b>0.901930</b><br>(0.0000) | <b>-0.042972</b><br>(0.0039)  | <b>0.981214</b><br>(0.0000) | <b>3.290254</b><br>(0.0000) |
| (1,1)            | <b>SGED</b>        | <b>0.054233</b><br>(0.0000)  | <b>0.036604</b><br>(0.0000)  | <b>0.129093</b><br>(0.0000) | <b>0.884521</b><br>(0.0000) | <b>-0.029983</b><br>(0.00474) | <b>0.979594</b><br>(0.0000) | <b>0.877055</b><br>(0.0000) |
|                  | <b>ESTD</b>        | <b>0.131606</b><br>(0.0003)  | <b>0.146169</b><br>(0.0000)  | <b>0.124051</b><br>(0.0000) | <b>0.894777</b><br>(0.0000) | <b>-0.037656</b><br>(0.0000)  | <b>---</b>                  | <b>1.063342</b><br>(0.0000) |
| <b>EGARCH</b>    | <b>SSTD</b>        | <b>0.045157</b><br>(0.0043)  | <b>0.068473</b><br>(0.0000)  | <b>0.218316</b><br>(0.0281) | <b>0.990111</b><br>(0.0000) | <b>-1.861725</b><br>(0.0000)  | <b>0.974975</b><br>(0.0000) | <b>2.01000</b><br>(0.0000)  |
| (1,1)            | <b>SGED</b>        | <b>0.049073</b><br>(0.0000)  | <b>0.021070</b><br>(0.0000)  | <b>0.012001</b><br>(0.2164) | <b>0.973459</b><br>(0.0000) | <b>- 0.238180</b><br>(0.0000) | <b>0.976664</b><br>(0.0000) | <b>0.876767</b><br>(0.0000) |
|                  | <b>ESTD</b>        | <b>2.589550</b><br>(0.0000)  | <b>0.022362</b><br>(0.0118)  | <b>0.138506</b><br>(0.0000) | <b>0.988636</b><br>(0.0000) | <b>-0.013369</b><br>(0.0720)  | <b>---</b>                  | <b>1.065231</b><br>(0.0000) |
| <b>APARCH</b>    | <b>SSTD</b>        | <b>0.044831</b><br>(0.05258) | <b>0.017666</b><br>(0.0510)  | <b>0.162967</b><br>(0.0000) | <b>0.899526</b><br>(0.0000) | <b>-0.119253</b><br>(0.1425)  | <b>0.971433</b><br>(0.0000) | <b>2.646986</b><br>(0.0000) |
| (1,1)            | <b>SGED</b>        | <b>0.043517</b><br>(0.000)   | <b>0.031323</b><br>(0.0000)  | <b>0.139839</b><br>(0.0000) | <b>0.882879</b><br>(0.0000) | <b>-0.028249</b><br>(0.0000)  | <b>0.971421</b><br>(0.0000) | <b>2.646986</b><br>(0.0000) |
|                  | <b>ESTD</b>        | <b>0.245612</b><br>(0.0000)  | <b>0.0345278</b><br>(0.0000) | <b>0.156738</b><br>(0.0002) | <b>0.763498</b><br>(0.0031) | <b>-0.047639</b><br>(0.0000)  | <b>---</b>                  | <b>1.084562</b><br>(0.0000) |

Table 3 presents parameter estimates for three asymmetric volatility models—GJR-GARCH (1,1), EGARCH (1,1), and APARCH (1,1) applied to Bitcoin returns under three different error distributions: SSTD, SGED, and ESTD. Across all models, most parameters are **statistically significant** ( $p < 0.05$ ), indicating that the models are well-specified and suitable for capturing Bitcoin's volatility dynamics. **ARCH effect** is positive and highly significant in all models which indicates **volatility clustering** in Bitcoin returns. The negative and significant leverage effect ( $\gamma$ ) in the result confirms the presence of the traditional "leverage effect" in the

financial assets bitcoin meaning that bad news (negative returns) causes a greater increase in volatility than good news (positive returns) of the same magnitude. This finding is line with the finding of **Bouivour & Selmi (2015)**, who found that Bitcoin's volatility is more influenced by negative shocks than positive ones, providing early evidence of the traditional leverage effect and contrary to the work done by **Baur et al. (2018)** and **Cheikh et al. (2020)**, whose research findings show an **inverse asymmetric effect**, where positive news increased volatility more than negative news, which they attributed to the herding behavior of uninformed investors.

**Table 4: Fitness Performance Selection based on LL, AIC and BIC for BTC**

| Volatility Models | Error Distribution | Log-Likelihood | AIC    | BIC    |
|-------------------|--------------------|----------------|--------|--------|
| GJR-GARCH (1,1)   | SSTD               | 4971.339       | 3.4488 | 3.4633 |
|                   | SGED               | 4970.922       | 3.4485 | 3.4630 |
|                   | ESTD               | 4971.893       | 3.4133 | 3.4365 |
| EGARCH (1,1)      | SSTD               | 4957.481       | 3.4392 | 3.4537 |
|                   | SGED               | 4958.853       | 3.4401 | 3.4546 |
|                   | ESTD               | 4973.432       | 3.4023 | 3.4209 |
| APARCH (1,1)      | SSTD               | 4944.939       | 3.4312 | 3.4477 |
|                   | SGED               | 4959.349       | 3.4412 | 3.4577 |
|                   | ESTD               | 4961.154       | 3.4210 | 3.4406 |

The empirical results of the fitness performance in table 4 indicate that the proposed Exponentiated Student-t Distribution (ESTD) outperforms the Skewed Student-t (SSTD) and Skewed Generalized Error Distribution (SGED) across all volatility models. Specifically, ESTD consistently yields higher log-likelihood values and lower

AIC and BIC criteria, confirming its superior ability to capture the distributional characteristics of Bitcoin returns, including skewness and heavy tails. This suggests that ESTD provides a more flexible and efficient error distribution for modeling financial time series volatility.

**Table 5: Forecasting Performance Evaluation of BTC using RMSE and MAE**

| Volatility Models | Error Distribution | RMSE     | MAE       |
|-------------------|--------------------|----------|-----------|
| GJR GARCH (1,1)   | SSTD               | 1.089788 | 0.7592027 |
|                   | SGED               | 1.090045 | 0.7588606 |

|              |      |          |           |
|--------------|------|----------|-----------|
| EGARCH (1,1) | ESTD | 1.071260 | 0.7567880 |
|              | SSTD | 1.090493 | 0.7584224 |
|              | SGED | 1.090290 | 0.7586021 |
| APARCH (1,1) | ESTD | 1.088635 | 0.7468298 |
|              | SSTD | 1.090511 | 0.7584088 |
|              | SGED | 1.090582 | 0.7583543 |
|              | ESTD | 1.090533 | 0.7580423 |

The forecasting performance result in table 5 shows that exponentiated student-t distribution (ESTD) consistently provides superior forecasting performance across all volatility models. Specifically, the GJR- GARCH (1,1) with ESTD model yields the lowest RMSE, while the EGARCH (1,1) with ESTD model produces the lowest MAE. This indicates that ESTD is more effective in capturing the heavy-tailed and asymmetric nature of Bitcoin returns compared to SSTD and SGED distributions). These findings in agreement with the findings of (Naimy & Hayek 2018). Abdelkarim & Laiche (2024) researched on the daily closing price of Bitcoin in

USD and found that EGARCH model performed other models, although he used normal distribution as error innovation but this contradicts the findings of Osterrieder & Lorenz (2017) on extreme value analysis of the returns of bitcoin exchange rate versus the USD for bitcoin and G10 currencies, particularly focused on their tail risk characteristics. Their result showed that bitcoin returns are much more volatile, much riskier and exhibit heavier tail behavior than the traditional flat currencies. It is important to note that the robustness of the error innovation employed determines how well these volatility models work (Panagiotidis et al. 2022).

**Table 6 : The Residual Diagnostic Result**

| Volatility Models      | LB Standardized Residuals P-value | LB Standardized Squared Residuals P-value | ARCH-LM p-value | Serial Correlation | ARCH Effect | Overall Diagnosis         |
|------------------------|-----------------------------------|---|-----------------|--------------------|-------------|---------------------------|
| EGARCH(1,1)<br>SSTD    | 0.4793                            | 0.9904                                    | 0.9826          | NO                 | NO          | Adequate / Well-specified |
| EGARCH(1,1)<br>SGED    | 0.3458                            | 0.7743                                    | 0.5730          | NO                 | NO          | Adequate / Well-specified |
| EGARCH(1,1)<br>ESTD    | 0.5341                            | 0.8341                                    | 0.7234          | NO                 | NO          | Adequate / Well-specified |
| GJR-GARCH(1,1)<br>SSTD | 0.4148                            | 0.8065                                    | 0.5627          | NO                 | NO          | Adequate / Well-specified |

|                         |        |        |        |     |    |  |
|-------------------------|--------|--------|--------|-----|----|--|
| GJR-GARCH(1,1)<br>SGED  | 0.0267 | 0.7872 | 0.5442 | YES | NO | Inadequate /<br>Residual<br>autocorrelation<br>remains |
| GJR-GARCH(1,1)<br>ESTD  | 0.5754 | 0.7100 | 0.6321 | NO  | NO | Adequate /<br>Well-specified                           |
| APARCH<br>(1,1)<br>SSTD | 0.3287 | 0.5224 | 0.3066 | NO  | NO | Adequate /<br>Well-specified                           |
| APARCH (1,1)<br>SGED    | 0.3959 | 0.6235 | 0.5547 | NO  | NO | Adequate /<br>Well-specified                           |
| APARCH (1,1)<br>ESTD    | 0.5212 | 0.9382 | 0.7231 | NO  | NO | Adequate /<br>Well-specified                           |

At 5 % significance level, the post-estimation diagnostic tests show that most volatility models adequately eliminated serial correlation and ARCH effects, as evidenced by insignificant Ljung–Box and ARCH-LM statistics ( $p > 0.05$ ). However, the GJR-GARCH (1,1)-SGED specification failed the residual serial correlation test ( $p = 0.0267$ ), suggesting possible model misspecification. Overall, the remaining models are considered statistically adequate for volatility estimation and forecasting.

## CONCLUSION

In this study, the performance of GARCH-type models; GJR-GARCH (1,1), EGARCH (1,1) and APARCH (1,1) under three distinct conditional distributions: Exponentiated Student t-distribution (ESTD), Skewed Student-t distribution (SSTD), and Skewed Generalized Error Distribution (SGED) is examined using the daily closing price of Bitcoin in dollars. The objective is to find out if the recently proposed Skewed Generalized Error Distribution (SGED) is more suitable in modelling the volatility of the daily closing prices of the Bitcoin. Based on the results from the data analysis using Python statistical software, the EGARCH (1,1) and GJR-GARCH (1,1) with ESTD error distribution is the overall best-performing model among those tested, based on these robust statistical criteria for modeling bitcoin. This selected model effectively captures the inherent volatility dynamics of Bitcoin returns while providing a strong statistical fit. For practical applications like risk

management and portfolio allocation, the forecasts from the model with ESTD error innovation is more reliable and can lead to better-informed financial decisions. In essence, ESTD error innovation results in a **more robust and accurate forecasting model** for Bitcoin volatility.

In comparison of the performance of asymmetric GARCH models estimation using the proposed innovation density with that of the existing improved skewed versions (skewed student -t and skewed generalized error distribution), the result of the analysis showed clearly the superiority of the ESTD error innovation over the improved versions. Therefore, these findings confirm that correctly specifying the error distribution is crucial for accurate volatility modeling and forecasting.

The diagnostic results indicate that all volatility models adequately captured serial correlation and ARCH effects, except the GJR-GARCH(1,1)-SGED model, which exhibited significant residual autocorrelation. Overall, the EGARCH and APARCH specifications demonstrated superior model adequacy and are considered suitable for volatility forecasting.

This study contributes to the growing literature on flexible innovation distribution on GARCH -type models by introducing an exponentiated heavy tail specification that improves the modeling of extreme financial returns without substantially increasing estimation complexity. Further research should extend this work to multivariate GARCH models, high frequency data, risk measures such as value at -risk and expected short fall.

**Conflict of Interest:** No conflict of interest to declare

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